Assembling the Meaning of *Probability*

James Duffy, Cecilia Moloney, and Terrance Quinn

First and Second Objectifications

A. James Duffy

My positioning focuses on the meaning of a single word, *probability*. As in the prior exercise,[1] my tasks are to (i) make explicit, to talk about, that which I am capable of talking about; (ii) to identify that which I am not capable of talking about; and (iii) to describe the view that results from developing what I take to be positional and reversing what I take to be counter-positional.

Probability is a word that rings familiar and commonly occurs in daily questions and assertions. "What is the probability that the baby will be a green-eyed girl?" "In all probability, we won't meet the deadline." "The probability of winning the lottery is quite low, but I will buy a ticket nonetheless."

Like so many words, *probability* can be used intelligently and common-sensically without understanding distinct, uncommon meanings that are the fruit of doing apparently trifling problems in twofold-attention. I do not know how many of the authors who have published in the leading journals[2] in the last 50 years have appropriated the basic insights that are my focus in this essay. And the relevant researches, interpretations, and histories have not arrived in the mail, so I am not in a position to pick out some good things and some not-so-good things from those journals.

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[1] In "Effective Dialectical Analysis," I highlighted the importance of implementing diagrams and heuristics in my thinking, planning, and teaching.

[2] The top journals include: *Journal of Statistical Software*, *Vital and Health Statistics*, *Annals of Mathematics*, *Annals of Statistics*, *Bioinformatics*, *Journal of the American Statistical Association*, *Biometrika*, *Annals of Probability*, and *Probability Theory and Related Fields*.

My efforts to appropriate the meaning of *probability* are guided by Lonergan's treatment of statistical heuristic structures in *Insight* and McShane's *Randomness, Statistics, and Emergence*.[3] Before trying to understand an uncommon meaning of *probability*, Lonergan proposes the following definition:

> Consider a set of classes of events, P, Q, R … and suppose that in a sequence of intervals or occasions events in each class occur respectively p_1, q_1, r_1 … p_2, q_2, r_2 … p_i, q_i, r_i … times. Then the sequence of relative actual frequencies of the events will be the series of sets of proper fractions, p_i/n_i, q_i/n_i, r_i/n_i … where i = 1, 2, 3 … and in each case $n_i = p_i + q_i + r_i + $ … Now if there exists a single set of constant proper fractions, say p/n, q/n, r/n … such that the differences $p/n - p_i/n_i$, $q/n - q_i/n_i$, $r/n - r_i/n_i$, … are always random, then the constant proper fractions will be the respective probabilities of the classes of events, the association of these probabilities with the classes of events defines a state, and the set of observed relative actual frequencies is a representative sample of the state.[4]

There are a couple of things that are noteworthy about the lengthy definition. First, it is technical and employs symbols to represent a sequence of events and series of proper fractions. Secondly, the definition has to do with counting the occurrences of events. Thirdly, it treats of a sequence of a series of events, not just one event or series.

Two exigencies follow from the two observations. There is an exigence to adequately classify or define events, with the stress being on recurrence-schemes rather than units, such as a population of genes.[5] A parallel and

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[3] Dublin: Gill and MacMillan, 1970. Hereafter I refer to this book as *RSE*.

[4] *Insight*, CWL 3, 81. See also Chapter 8 "Foundations of Statistics," in *RSE*, pp. 149–169.

[5] McShane elaborates on the crucial difference in *RSE*, Chapter 10, "Emergence and Recurrence Schemes." Beyond the question of recurrence schemes, there is the question of their emergence and survival. Human survival depends on intelligent management of flexible circles of cycles of schemes of recurrence, where the word "intelligent" includes the child's "What might that be, mommy?" and the word "circle" is metaphorical, not literal or physical. Our challenge is to shift probabilities, e.g. of "genuine leisure" for our great-grandchildren, from the product of fractions to the sum of fractions. See further my commentary on "the probability of the combination of events constitutive of the scheme leaps from a product of fractions to a sum" (CWL 3, 144) in the article cited in footnote 8 below.

easier challenge is to move beyond descriptive talk of "wealth," "poverty," and "profit" that plagues the massive analysis of statistics in Piketty's *Capital in the Twenty-First Century* to an elementary clarity regarding the basic terms and relations of sane economics.[6] A second exigence revolves around the issue of determining how large a sample is sufficient enough to be a representative cluster. Applying statistics to small samples, or even to an individual event, is misleading.

Statistical inquiry is concerned with nonsystematic processes. It focuses on random differences in aggregates of events that are juxtaposed temporally or spatially. Consider the following five series:

(i)   A, B, C, D, E, F, G, H, …

(ii)   H, T, T, T, H, T, H, H, …

(iii)   O, T, T, F, F, S, S, …

(iv)   M, T, W, T, F, S, S …

(v)   1, ½, ⅓, ¼, ⅖, ⅙, 3/7, ⅛, …

The first series appears to follow a type of 'formula' and a grade schooler could readily fill out the series. The third and fourth series also follow a type of formula. They are ordered series whose progression can be worked out once you understand the formula.[7]

What about the second and fifth series? Like the other three series, they are spatially juxtaposed marks on a page. Are they ordered? Let me refine the question by specifying that in the second series "H" and "T" refer to heads and tails and by stating that the fifth series is the same sequence

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[6] A concise introduction to basic economics terms and relations is presented by Philip McShane in the preface to the third edition of Economics for Everyone: Das Jus Kapital (Vancouver: Axail Publishing, 2017), iii–v. In *Piketty's Plight and the Global Future* (Vancouver: Axial Publishing, 2014), McShane asks, "How much of Piketty and Galbraith and their collaborators is worth recycling?" (70) He writes in the footnote about a needed "shift to adequate categories of collecting and sifting data and facts" (note 114, p. 70). I elaborated my position regarding the basic terms and relations and adequate categories in "Minding the Economy of *Campo Real*," *Divyadaan: Journal of Philosophy & Education*, vol. 29, no. 1 (2018), 1–24.

[7] A hint to help figure out the formula of the third series is to line it up with the fifth series. A hint to help figure out the formula of the fourth series is "9 to 5," a song written and originally performed by the American country music singer Dolly Parton.

expressed differently. Is there a rule or formula for filling in the three dots in these two series as there is for the other three series?

If I toss an unbiased coin ten times, and each time it is heads, you would begin to wonder. If I do this five consecutive times, that is, heads results fifty times in a row, you would ask: "Say, James, how do you do that?" Your antecedent expectation is that the number of heads in each series of ten should oscillate.[8]

Take fifteen minutes to flip a penny or dime fifty times, divide the flips into series of tens, and write out the actual frequency of heads for each of the five series. The fraction can and often will differ from one-half. The full, basic insight into insight is a matter of catching myself in the intelligent act of grasping a regularity in the relative actual frequencies by positively abstracting[9] from what is random, observe a clustering, and identifying the center about which events oscillate. There is nothing a priori or pejoratively abstract about doing this, as probabilities are hypothetical; they call out for verification.[10]

I am not in a position to say if Lonergan's discussion of the meaning of *probability* is an advance over others put forth in the period 1654–1957, i.e.,

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[8] "*El azar, la probabilidad emergente y la cosmópolis*," (Randomness, Emergent Probability, and Cosmopolis), *Revista de Filosofía* (Universidad Iberoamericana) 135 (2013), p. 317, my translation. In the next paragraph I comment on a cocktail party where "the majority of the people are gathered in the kitchen. Spontaneously you ask yourself 'Why? Is that where the appetizers and drinks are? Is somebody telling a joke?' The questions manifest a spontaneous expectation that the groupings of people should be random, and if not, there must be a reason. If the majority or all people are gathered in the kitchen, there must be a reason to explain the congregation" (317–18).

[9] See the long paragraph beginning "Fourthly," (*Insight*, CWL 3, 40) as well as the paragraph beginning "However," on the bottom of CWL 3, 87.

[10] "The theory of probability is not 'just like mathematics': it is more closely concerned with the concrete than classical mathematics. Unless the techniques of application play a precise role in the development of the theory, the theory is liable to generate pseudo-techniques of application which diverge from empirical requirements." *RSE*, 164. The solution to the basic problem of selecting representative samples depends not just on the development of statistical method, "but also on the general knowledge of individual investigators and on their insights into whatever specific issue they happen to be investigating." CWL 3, 82.

to what extent his implicit definition[11] is a breakthrough. But McShane suggested 50 years ago that a possible scientific advance of the basic insight into the meaning of *probability* is to consider how we might go about studying total history, an extremely complex manifold of events in which developments and set-backs can and do occur. Think of the expression "What a state we are in!" Sixty years ago F.M. Fisher sought to produce "a rather grandiose picture of history"[12] by defining a *state of nature* as being a point in *m*-space (m is the number of independent variables, excluding calendar time). He specifies time as *n* discrete moments of world process. Finally, he proposed a Markov matrix[13] to approximate a view of total process, world history. In this approach to total history, "Toynbee's *Study of History*" could be regarded as something of a "reduction of the historical process to a very few variables and very large subdivisions."[14]

Where does my view lead? What is my hope for "the story of *probability*"? I foresee in hope and fantasy the basic problems—classifying satisfactorily and selecting representative samples—being taught at a basic level by tossing pennies and observing clusterings of students in classrooms or cafeterias, and by "encouraging in them the conscious occurrence of the intellectual events that make it possible to know what happens when probability is grasped."[15] This small sub-task of total education will reverse any tendency to associate statistical method with number crunching by those hoping to apply statistics in areas as diverse as education, fund management, and holistic medicine. Total educators and

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[11] An implicit definition prescinds from what rings familiar about the terms and focuses on relations, which allows for the inclusion of isomorphic cases. See CWL 3, 37 and *RSE*, 165–69.

[12] "On the Analysis of History and the interdependence of the Social Sciences," *Philosophy of Science*, vol. 27 (1960), p. 150, quoted in *RSE*, p. 237.

[13] An *n* x *n* matrix in which all entries are nonnegative and the sum of each column vector is equal to 1. For example, $A = \begin{bmatrix} 1/2 & 1/3 \\ 1/2 & 2/3 \end{bmatrix}$

[14] "On the Analysis of History and the interdependence of the Social Sciences," p. 156, quoted in *RSE*, p. 237. See also Lonergan's comments about "differentials of what has flowed" when "asking about the meaning of history." *Essay in Fundamental Sociology*, 19–20. Recently McShane has commented on the Fisher-Markov network of probability-schedules and a possible sublation of Fisher-Markov in the philosophy of history. See *The Future: Core Precepts in Supramolecular Method and Nanochemistry* (Vancouver: Axial Publishing, 2019) notes 63–64 (p. 91), note 90 (p. 74), and the second paragraph on page 99.

[15] *Insight*, CWL 3, 82.

educated will also be aware that the meaning of *probability*, like the meaning of *fire* or *ellipse*, is a dialectic-genetic history.[16]

A second, related development to look forward to is an increase in thinking about big numbers and long periods of time. The climate crisis and, in some parts of the globe, the water crisis, is forcing us to think thusly.[17] Those implementing statistical method are facing forward, fantasizing and anticipating Poison events, break-outs from the gross ugliness of contemporary culture that is censored by the axial superego of little human beings.[18] More time will be spent in secondary and higher education thinking about the future, even asking what might appear to be irrelevant questions, such as: What might be valuable statistics in 2020 for planning a more livable human life in 3020?

Another result of my view of insight into statistical inquiry and the inquirer is a reversal of empirical scientists copping out of luminous practice by saying that philosophers are the ones who bother with "true meanings," as well as a reversal of philosophers copping out by saying we can get by just fine without implementing statistical heuristic structures, without distinguishing pejorative and positive meanings of *abstract*, and without distinguishing classical and statistical types of abstraction.

Finally, at a future date that I find difficult to specify, the phrase "long-term normal," currently used to describe the distribution of stock returns, will be used to talk about the emergence of the positive Anthropocene, a time when high schoolers will self-understand that there was nothing

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[16] On the ongoing story of *fire*, see "'MacIntyre and Lonergan' Revisited," *Journal of Macrodynamic Analysis*, vol. 12 (2020), 82–84. On the ongoing stories of *ellipses* and *probability* theory, see *RSE*, 156–160.

[17] See further my comments about Tarja Ketola vaguely leaning into the future in her essay "Climate Change Immigrants or Refugees of the Anthropocene—Adapting to or Denying Climate Change?" (*Sustainability and Peaceful Coexistence for the Anthropocene*, ed. Pasi Heikkurinen [Routledge, 2017], 31–48) in "Learning From History to Plan Migration" (available at: https://www.anthropositivecene.org/2019/01/18/3-learning-from-history-to-plan-migration)

[18] "Just as the little birds know that twigs are good for building nests and the little lambs know that wolves are bad, so little human beings develop a cogitativa about good and bad; it reflects their childish understanding of what papa and mamma say is good or bad and in adult life it can cause a hell of a lot of trouble." A letter from Lonergan to F. Crowe, December 27th 1955. Quoted in P. McShane, *Humus* 2: *Vis Cogitativa*: Contemporary Defective Patterns of Anticipation, available at: http://www.philipmcshane.org/humus.

normal about the way financial gurus used the phrase "long-term normal" in the twentieth and twenty-first centuries.

B.   Cecilia Moloney

Task 1: A First Objectification

The dynamic structure of human knowing "goes beyond" data, intelligibility, and known truth, to the truth and being still to be known.[19] Within the limits of empirical science, the dynamic structure of human knowing aims "at a complete explanation of all phenomena."[20] In Chapters 1–4 of *Insight*,[21] Lonergan employs the pedagogical technique of the moving viewpoint as he develops the unification of classical and statistical laws of empirical science. While Lonergan's examples in Chapters 1–4 emphasize mathematics and the natural sciences, it is clear that Lonergan is building towards the wider perspective of later chapters of *Insight*.

In fact, Lonergan's interest is human progress. From the beginning of *Insight*, he builds towards a way of understanding what he calls "world process in its concrete historical unfolding."[22] In his discussion of probability and statistical laws of the nonsystematic in Chapters 2 and 3, he develops the viewpoint from which we can seek to understand world process, in order to better enable human progress. Within Dialectic, the contribution is to "[present] an idealized version of the past, something better than was the reality."[23] What I think Lonergan meant by this sentence is that in Dialectic, we seek a more solid ground to stand on while moving towards the future and the solution of problems, whether these problems are new problems that the future may bring, or current problems that are the products of the past or vestiges of past problems that continue to linger.

Thus, Lonergan can offer the following summary of the first four chapters of *Insight* by noting his method of the moving viewpoint:

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[19] Lonergan, B.J.F. (1988). *Collection*, 2nd ed. Ed. F.E. Crowe and R.M. Doran. Toronto: University of Toronto Press (CWL 4), p. 211

[20] Lonergan, B.J.F. (1973). *Method in Theology*, 2nd ed. (London: Darton, Longman and Todd), p. 129. All page numbers in this exercise for *Method in Theology* reference this edition. Hereafter *Method.*

[21] Lonergan, B.J.F. (1992). *Insight: A Study of Human Understanding*. Eds. R.M. Doran and F.E. Crowe. Toronto: University of Toronto Press (CWL 3). Hereafter *Insight*.

[22] *Insight*, p. 115.

[23] *Method*, p. 251.

As we began by inviting the reader to grasp the intelligibility immanent in the image of a cartwheel, so now we are inviting [her] to perform again the same kind of act. The only difference is that, for the image of the cartwheel, [she] now must substitute the main features of the universe of our experience.[24]

And what are the main features of the universe of our experience? Substantially, this universe of experience changes. It changes systematically. It also changes nonsystematically. How are these two modes of change to be reconciled? Can they both go together? Can they both be understood? Is one dominant? Is one illusory? Such questions have been asked in the past, and still are asked. Lonergan outlines the framework within which to answer these and other questions via the intelligibility of the systematic and nonsystematic, and the complementarity of classical and statistical laws both in the operations of the knowing as well as in what is known.[25]

Task 2: A Second Objectification

I structure my second objectification around examples of global contexts that are significant and may continue to be significant within "world process in it concrete historical unfolding". I refer to 1) climate change, and 2) the development and implementation of artificial intelligence (AI).

Both contexts are highly complex and difficult to understand in their entirety and require diverse scientific or technological knowledge and expertise to understand and to develop or guide. Hence, it is challenging to predict their unfolding impact on human living and on our environment. They also generate debate, on scientific details, and also on their likely impact; the latter leads at times to emotionally-charged conflict, even to entrenchment in ideological positions, notably with respect to "what to do" questions. As such, both contexts cry out for functional collaboration, and in particular, for Dialectic to get at and sort out the roots of conflicts. Finally, both contexts must be understood, in part, via their nonsystematic data. Hence, Lonergan's writings on the meaning of probability and on the complementarity of classical and statistical laws can provide keys for understanding and developing positions and reversing counter-positions.

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[24] *Insight*, p. 151.

[25] *Insight*, Chapter 4, Sections 1 and 2.

Due to space limitations, I will offer only brief insights and suggestions of how Dialectic might enable us to understand the past better than it has been, and so position us for a brighter future of authentic human living.

Example 1: Global Climate Change

Climate change was a key theme that was debated in the 2019 federal election in Canada[26], framed largely in terms of "what to do" about the climate crisis. However, such conversations are often premised on unspoken assumptions about how serious climate change is. Indeed, one might claim that climate change is a complex context requiring an understanding of both classical laws and statistical laws.[27] Most citizen electors—as well as most scientists—lack the full knowledge needed to truly understand climate change. Further, better collaboration is needed so that diverse knowledge, expertise and perspectives can be factored into decisions and actions. Within Dialectic, I think there are root differences that can be brought to light in order to enable collaboration towards "cumulative and progressive results."[28]

I will note only two sources of difference based on flaws in understanding, in one case of classical law, in the other case of statistical law.

According to the late physicist Al Bartlett, "The greatest shortcoming of the human race is our inability to understand the exponential function."[29] Climate change started slowly, and so it has been possible for many of us to dismiss expert predictions based on nonlinear models of change, and rather to intuit a more comfortable future based on extrapolating linearly from small observed changes. Thus understanding even classical models of climate change can be challenging.

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[26] Election day was October 21, 2019.

[27] Climate change also challenges us—as does the rise of artificial intelligence—to better understand ourselves as human beings. A favourite quote is mine, one that started me on my study of Lonergan's works, is, "How can we be clear about our relationship with the natural world until we have at least tackled the question of what it is to be a human being." Margaret Goodall and John Reader, "Environmentalism as the question of human identity," in *The Earth Beneath: A Critical Guide to Green Theology*. Ed. I. Ball, M. Goodall, C. Palmer and J. Reader, London: SPCK, 1992, p. 47.

[28] *Method*, p. 4.

[29] Al Bartlett, "Arithmetic, Population, Energy – a talk," available at https://www.albartlett.org/presentations/arithmetic_population_energy.html

Second, it is challenging to deal with the nonsystematic and with statistics. For example, we might hear someone say, "Sure, this was a bad storm, but the weather has always been variable. How can you say the events are more extreme now?" Such questions can be settled by statistical law and the intelligibility immanent in observed data, such as in averages tracked over time, etc. However, many people dislike statistics—even those who have taken courses in statistics—and so many prefer to skip ahead of trying to understand the statistics of climate change and to focus instead on the "what to do about it" question.

For these and other reasons, we are challenged to dwell at Dialectic to understand the past and to develop a comprehensive viewpoint. Lonergan's viewpoint that unifies classical and statistical law, and enables both aspects of science to work together, would assist scientists as well as citizens to better understand the complex context of climate change.

Example 2: Artificial Intelligence (AI)

Lonergan wrote in *Insight* that "the profound significance of statistical laws is coming to light."[30] This sentence, written over 60 years ago, remains true today, and especially with the recent rise of AI technology. The example of AI is pertinent to this Dialectic exercise on Lonergan's definition of probability because AI technologies use observed data to train their algorithms by searching for patterns or correlations in the training data.[31] Assumptions of statistical stationarity are then implicitly invoked so that a trained AI algorithm can select the most likely pattern or correlation to apply to new data. With recent increases in accuracy and speed, fueled by big data, AI is poised for deployment in many sectors. Indeed AI is now viewed almost as a universal solution.

However, AI is an approach that is reliant on statistical laws; classical models are used in neural networks, but their selection is based on what makes the patterns and correlations most predictive, rather than on the notion of classical law as explanatory.[32] Thus, I think that AI may fall into the trap noted by Lonergan:

> But if this new movement [of statistical laws] is not to degenerate into
> the old talk about what commonly happens, it must retain its contact

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[30] *Insight*, p. 135.

[31] Floridi, Luciano (2019). "What the Near Future of Artificial Intelligence Could Be," *Philosophy and Technology*, 32, 1–15.

[32] Worth noting is that classical laws may guide other components of technologies that AI would be contributing to.

with the empirically established precision of classical formulations. For statistical laws are of no greater scientific significance than the definitions of the events whose frequencies they determine; unless these definitions are determined scientifically, statistical thought lapses into prescientific insignificance.[33]

I include this quote not to deny the value of AI and the contribution that it could make to human progress, but to underscore the need to be fully aware of what AI is and how it works. I fear the latter are not well understood by many, not even by scientists and engineers, who increasingly rely on statistical law, and in some cases without adequate understanding of, or advertence to, the assumptions behind their reliance.

Moreover, a reliance on statistical law opens up the possibility for bias, conscious or unconscious, in the sampling that selects the training data, with the result that the extracted patterns and correlations would tend to favour the bias, encoding it as part of "reality".

In summary, understanding the reliance of AI on observed data and on patterns and correlations in the selected data is crucial for scientific and ethical critiques of AI. Lonergan's worldview is key to moving forward towards developing positions and reversing counter-positions within Dialectic, and for the Foundations needed for the development and deployment of human-respecting and progressive technologies of the future.

C. Terrance Quinn

Sections 1 and 2 are on personal sources, in what I am provisionally calling "empirical probability" and "adverbial probability," respectively. Section 3 is on counterpositional sources. Keeping my focus on *probability*, Section 4 is on "the view that would result."[34] Section 5 is on some reversal strategies for problems identified in section 3.

1. Empirical probability[35]

Ecologists claim that 200–2000 (up to 0.1%, 1000 times the normal background rate) of the world's species of organism have gone extinct each

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[33] *Insight*, p. 135

[34] *Method in Theology*, 250; CWL 14, 235.

[35] My intended "first readers" are my two collaborators in this exercise in (proto-) dialectic, Cecilia Moloney and James Duffy. All three of us have backgrounds that include some experience with modern mathematics,

of the last several years[36]. Presently, then, *empirical probability* of extinction in a year is approximately 0.1%.

What do I mean by empirical probability?[37]

In the example, we find two possible outcomes, extinction or non-extinction. In my experience in employing statistical method in scientific inquiry,[38] we attempt to identify (ranges[39] of) possible events or occurrences. Each range is called a *sample space S*.

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applications and "self-attention" (dynamics of knowing and doing, see Appendix A in CWL 18 [*Phenomenology and Logic*], 322–23). In this project, we are attempting to work within a self-imposed limit of "1300 words +/-." That word limit resulted in a density of expression. (Excluding section titles and footnotes, the main body of text is approximately 1610 words.) Functional dialectic is a remote future achievement. Toward that possibility, I hope that readers will be interested in the results of our preliminary experiment. Trying to serve two purposes, then, I structure the paper as follows: the main body of text is brief description, where I move along the surface of my positioning. More advanced personal sources are pointed to in footnotes. I have decided to let the forthcoming challenge of "third objectification" (we don't yet know how that will work) help determine to what extent more detailed "self-revelation" might be called for. Perhaps I will need to attempt more detailed description of "layerings" in my understandings in empirical probabilities (in elementary instances; in my grasp of Normal, Poisson or modern distributions). See note 37. I could see that approach being the basis for a differently focused "first and second objectification."

[36] See, for instance, *Center for Biological Diversity*, https://www.biologicaldiversity.org/programs/biodiversity/elements_of_biodiversity/extinction_crisis/. Accessed October 17, 2019.

[37] In what follows, I draw on my experience in the mathematical sciences, and philosophy of science. (See, e.g., Quinn, *Invitation to Generalized Empirical Method* [Singapore: World Scientific Publishing, 2017]). In 2008-2009, I learned something of sequence alignment methods. Our team drew on and verified results in statistical data obtained from reactions in laboratory experiments. We were studying the biochemistry of *ribonucleoside hydrolase* in *E. Coli* (*rihC*). A segment of one of a plethora of known reaction pathways of *E. Coli* is: *Nucleoside + H2O → ribose + purine or pyrimidine*. The one-celled organism *E. coli* is an amazingly sophisticated one-celled aggreformic entity. The compound *ribonucleoside hydrolase* also is aggreformic, defined within the vast physico-chemical "eco-system" called biochemistry.)

[38] I mean this inclusively of human sciences now and to be.

[39] The "type" of event or occurrence is an empirical problem. It is sometimes mainly a descriptive classification. For instance, in card games, an event can be

For an aggregate of events or occurrences A from S, $m(A) \geq 0$ is the total number (or measure)[40] of events or occurrences in A; while $m(X|A) \geq 0$ is the number (measure) of events or occurrences of "type"[41] X in the aggregate A.

The key insight: In cases where $m(A) > 0$, if it is verified that relative actual frequencies $m(X|A)/m(A)$ "cluster"[42] about some p, $0 \leq p \leq 1$, then p (in this article, denoted $Pr(X)$) is the (provisional[43]) *empirical probability* of X.

In anticipation of section 3, I draw attention to two special cases, namely, $p = 0$ and $p = 1$. For me, it is evident-and-self-evident (object-oriented-and-subject-oriented) that the meaning of $Pr(X)$ does not change according to its value, $0 \leq p \leq 1$. Relative actual frequencies $m(X|A)/m(A)$ cluster about *0* if and only if $Pr(X) = 0$. There is a similar result for when relative actual ratios cluster about unity, in which case $Pr(X) = 1$. In other words, empirical probability being zero (or unity) does not mean "X certainly never (or certainly always) occurs." Indeed, if in certain

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"Ace of Diamonds." We make progress in distinguishing "explanatory genera and species." However, "genera and species" also is a heuristics. I take as basic, but remote to my and our present achievement, the densely expressed heuristics given by Lonergan, *Insight*, CWL 3, 489, 609–610. For some personal sources, see note 37.

[40] In the simplest cases, one "counts" events and occurrences. What is "counting"? Explanatory identification (of "counting") is a remote future achievement for human sciences. For instance, see notes 37, 38, heuristics indicated in note 39, concluding paragraph to section 2, and note 58. A 20th century development, "measure theory" is mathematics, while a "probability space" is when the set of all possible mathematical events or occurrences is of unit measure. Whether or not a "probability space" is part of understanding (patterns in) events and occurrences of actual process is an empirical problem.

[41] See note 39.

[42] The expression "cluster" is descriptive of what is being done in applied mathematics. I do not use the expression "ideal relative frequency," because the name is vulnerable to being taken up in "philosophical debate." The structure includes: a provisional measure m, some way of distinguishing events or occurrences X in aggregates A of events or occurrences from S. Cluster points (in relative actual frequencies) are (usually) in terms of the standard metric for the real interval $0 \leq p \leq 1$. Questions often include whether or not divergence from p is "random"; the measure m and sample space S fit the process being investigated; and whether or not there is evidence for as yet unaccounted for "systematic" factors influencing outcomes.

[43] See notes 39, 40, 41 and 42. The inquiry is empirical.

circumstances, either *X* never occurs or aggregates *A* consist only of events or occurrences of type *X*, then statistical method is not needed.

2. "Adverbial probability"

I make use of the example about extinction rates:

(i) What ecologists claim about global extinction rates is probably true. In other words, we find "probable judgements" about what is so. Ecologists also make forecasts. For instance, based on prior ratios, on the fact that the total number of species remains large, and on the fact that there have been no major changes in global strategies being implemented, there is no reason to expect statistically significant changes in next year's extinction rates. Within error intervals, global extinction rates next year probably will be approximately 0.1% (or more).[44]

(ii) Concerned groups have been developing (probably) feasible plans to address the problem of global sustainability.

From these observations, it is evident that, in addition to empirical probabilities $Pr(X)$ $(0 \leq Pr(X) \leq 1)$, broadly speaking[45], there are two other common "uses" of the word 'probability':

(i) In a first mode, 'probably' is an adverb for the quality of one's judgment that follows an 'Is it so?' question or a 'Will it be so?' question.

(ii) In a different mode, 'probably' is an adverb for the quality of one's judgement that follows an 'Is it to be done?' question.

What I am calling "adverbial probabilities" also are "empirical," but not in the sense of being "(numerical) empirical probabilities in scientific inquiry" (Section 1). They are empirical in the sense that "estimating," "forecasting," "betting," "predicting"; "identifying good plans," indeed all judgements, are probable; and are given, are data, are experience.

My present view of 'probability,' then, is mainly descriptive and evidently thin. In scientific inquiry, we ask 'What is *X*?' We also inquire after frequencies and relative actual frequencies of *X*. But an understanding of an empirical probability also is an *X*. And so, I envisage (future) ongoing

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[44] The claim is empirical. Verification is by appealing to data and by reaching further judgements of fact about aggregates.

[45] See note 39.

progress in finding and "organizing"[46] "genera and species"[47] of emergent empirical and adverbial probabilities, as well as emergent "genera and species" of empirical probabilities of emergent genera and species of empirical and adverbial probabilities.[48]

3. Some counterpositional sources

I draw attention to what I perceive to be four not independent groupings of currently influential counterpositional sources:

(i)   In the tradition of undergraduate and graduate textbooks on statistics and probability, as well as in contemporary philosophic debate, different uses of the word 'probability' regularly are combined and confused. For instance, "certainty" regarding particular aggregates often is identified with "unit probability"; and "impossibility" with probability zero.[49] In another set of errors, mathematical limit (including laws of large numbers) are (mistakenly) identified with *Pr(X)* (which is empirical and allows for random differences, even when aggregates are large). Not surprisingly, there result endless technical errors and apparent paradoxes. There is, for instance, a growing literature that includes silly but solemn claims about 'the universe' being a 'multi-verse.'

(ii) Errors mentioned in (1) are not merely technical errors. They are counterpositional. For, they emerge from and appeal to criteria obtained from conceptual or imaginary models about "probability." Arguments are self-evidently inconsistent with experience, which includes our knowing and doing that are our sources of meaning for our use of the word 'probability.'

(iii) "Wall Street" and schools of finance are candid about the fact that investment strategies are for gambling.[50] Those applications of probability theory are counterpositional in several fundamental

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[46] I expand on this in the last paragraph of section 4.

[47] See note 39.

[48] This will be possible within a "balanced" or "generalized" or eventually "(adequate) empirical method."

[49] See the last paragraph of section 1.

[50] The goal is to earn profits for shareholders and other investors. Applications are of Monte Carlo methods and other results from modern stochastic analysis.

ways, which I cannot go into here. However, one way that directly pertains to this article is the mistake of identifying adverbial probability (about investment strategies and economic policies) with empirical probabilities (verified in aggregates of prior investments and economic outcomes).[51] Problems that follow include: computer algorithms used to allegedly quantify "the human decision-making process" in finance; and, more generally, views and applications of so-called "artificial intelligence."

(iv) In Lonergan Studies, so far, there is little evidence of operative understanding of empirical probability in elementary, classical or modern contexts. There may be few exceptions but (with approximately unit empirical probability) the tradition has been effectively blocking scholars interested in Lonergan's work from making entry into scientific contexts. This is "contra-positional," in the sense that the approach is contrary to Lonergan's explicit counsel repeated throughout his opera omnia. The approach also is counterpositional. It fails to break free of old-style methods. It continues to admit linguistic competence and extra-scientific conceptual constructs as sufficient basis for modern progress in philosophical and theological reflection. As a result, Lonergan studies has been promoting ongoing discussion about "occult entities" with names such as 'event,' 'occurrence,' 'schemes of recurrence,' 'probability,' 'emergent,' 'probability of emergence,' and 'finality.'

4. With *probability* my focus, my "view that would result"[52]

It is well known that world process includes vast ranges of "(statistical) schemes of recurrence."[53]

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[51] Probability methods in contemporary finance are counterpositional in other ways, too. For instance, the "*X*'s" of contemporary economic models, as well as alleged economic goals, are incompatible with the verifiable fact that in any economy there are five functions and two flows; that there are human needs; and that there is the problem of global sustainability (all types).

[52] See note 34.

[53] In paragraphs below, I draw attention to why the word 'statistical' is needed. Schemes of recurrence are found in, for instance, education and culture, economics and commerce, global ecosystems, and beyond. The effort to identify schemes is a major part of ongoing scientific inquiry. There is, for instance, progress in identifying multi-stage and multi-species life cycles, predator-prey

On a first approximation, a scheme of recurrence can be described as a regularly occurring cyclic sequence of events and occurrences $A(1) \rightarrow A(2) \rightarrow \ldots \rightarrow A(k)$ $((A(k) \rightarrow A(1))$ (events and occurrences $A(i)$, "transition" events and occurrences $A(i) \rightarrow A(i+1)$, and subsequent events and occurrences $A(i+1)$, $i = 1, 2, \ldots, k$). But very few known schemes of recurrence have the structure "If $A(i)$ occurs then $A(i+1)$ occurs;…; and if $A(k)$ occurs, then $A(1)$ occurs, $i = 1,2, ..., k.$" More often they are of the form: *"If a sufficient number of $A(i)$ occur, then some number of transition events or occurrences $A(i) \rightarrow A(i+1)$ probably will occur; …; and so on."* A scheme of recurrence, then, is not merely a cyclic sequence of statistically independent events or occurrences. Key to the functioning of a scheme of recurrence is that, at least for a time, there are strictly positive conditional probabilities. In known schemes, we also find non-negative *conditional probabilities* for transition events and occurrences $A(i) \rightarrow A(j)$, $i \neq j = 1, 2, \ldots, k$. These can influence frequencies of events and occurrences of the central scheme. In other words, known schemes of recurrence are (i) statistical and (ii) part of processes that are Markov-like[54].

Schemes of recurrence are verifiable. But, evidently and self-evidently, they are neither events nor occurrences. And so, for a scheme of recurrence, empirical probabilities of events and occurrences (and for particular sequences of events and occurrences, including cyclic sub-sequences called *schemes of recurrence*) are (not products of probabilities but, rather) sums (of products) of conditional probabilities.[55]

As is verifiable in known instances of emergence, growth, development, decline, recovery, failure to recover, and extinction (including our own knowing and doing, lives, history), total process (of which we are a part – basic position, W2[56]) is fundamentally "dynamic." Total process,

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patterns, intermediates in pathways of multi-valent and often reversible chemical reaction "pathways" in global ecosystems, astronomical events and occurrences, economic supply chains, patterns of education, exploitation and other groupings of human collaboration, for better or for worse.

[54] I say "Markov-*like*" because actual schemes change whereas a Markov process, as such, has a fixed sample space and constraints on conditional probabilities. Note also that known schemes of recurrence are substructures of "combination schemes of recurrence" where an ith stage is of the form $A(i) = (A(i1), A(i2), \ldots, A(ik_i))$, with the $A(ij)$ being events or occurrences.

[55] This is a feature of Markov processes.

[56] Philip McShane, *Prehumous* 2, "Metagrams and Metaphysics," W2, available at: http://www.philipmcshane.org/prehumous.

then, is not Markov as such. But for subsets and time intervals of relative stability, processes are "Markov-like." In total process, emergent and changing events, occurrences and schemes are verifiably mutually dependent in vast layerings and sequencings of statistical dependence.

Evidently-and-self-evidently, we-and-with-the-universe are an emergent "*process-in-progress*."

What is my view of *probability* that results?

Empirical probability is the "science that focuses on *act* – past, present, possible and probable." In that science, progress is evident-and-self-evident in our ongoing efforts to identify events and occurrences (acts), schemes of recurrence of events and occurrences, relative actual frequencies of events and occurrences, cluster points in relative actual frequencies of events and occurrences called *empirical probabilities*, and that way also[57] empirical probabilities of schemes of recurrence. Total process is no "mere aggregate" but is, rather a vast dynamic "organic assembly." And, advances in the science of empirical probability contribute to progress in "onto-genetic" and "phylo-genetic" systematics of that "organic assembly" that is the "body of history."[58]

5. Some reversal strategies for 3(i)- 3(iv)

(i) Sciences, philosophy of science and theology need to make progress in adverting to our dynamics of knowing and doing. This will provide a basis for a needed control of meaning in implementing probabilities, and more.

(ii) Economics needs to break free of its current mode of conceptualist "model building," untenable in any serious science. Then economics will be able to make progress in obtaining statistical analyses of two flows and five functions of events and occurrences. This will be part of progress in benevolent and effective management of global two-flow economies.

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[57] See second and third paragraphs of this section 4.

[58] What I have briefly described is part of a "larger" conversation. For instance, there are empirical probabilities of genera and species of gift. I will not attempt to develop this here. I do, however, refer back to my contribution to an earlier exercise, the first one in this volume, where I attempted to provide first and second objectifications regarding "comparison" and the "mystical body of Christ."

(iii) Lonergan studies needs to promote scientific understanding; and also needs to implement strategies pointed to in 5(i) and 5(ii), above.

Third Objectifications

A. James Duffy

The third and final objectification of horizon, is the assembly of the above text written by the three of us, the development of positions, and the reversal of counterpositions. As in the first two objectifications, I am doing my best to manage word limits. I am aware that we are trying to do something untried, trying to set up a new model of encounter in the hopes of presenting an idealized version to foundationers.

Some positional elements that, I believe, merit further development. The symbolization in Quinn and in Duffy, is normative for positioning probability and the meaning of *emergent probability*. All three authors are concerned about how *probability* is linked to concerns about the concrete good, e.g., what we humans might do about changes in the climate, economic model building, and the need to become better teachers of statistical methods. Moloney pulls in classical methods and refers to the canon of complete explanation. She also refers to the moving viewpoint from which *Insight* was written and notes that Lonergan's concern was with human progress in world process. Quinn comments on schemes of recurrence, noting that they are statistical and Markov-like, while Duffy refers to F.M. Fisher's proposal to incorporate Markov matrix to approximate a view of total process.

There are also positional elements regarding teaching (communicating) basic insights that merit further development. In the idealized past/future better than it was, the questions that Moloney poses regarding the systematic and nonsystematic will be taught in the "new" mode of twofold attention in which I am better aware of a possible gap between Lonergan's understanding of "world process in its concrete historical unfolding," my understanding, and my student's understanding. Duffy relates this to encouraging students to appropriate "the conscious occurrence of an intellectual event." Quinn uses the phrase "evident and self-evident" a number of times to highlight twofold attention and twofold understanding. The task is for teacher and student alike to strive for something more than technical competence. As Moloney notes, some people simply dislike learning statistics, which is a form of self-dislike that is part of the social surd and bigger than any self.

Another positional element that merits further discussion and development are the failure not just to (self-)understand *probability*, but the ongoing conflation of linguistic competence with the skills and habits to effectively intervene in world process. Both Duffy and Quinn identify this as a challenge that knows no disciplinary boundaries. It is a challenge to one and all, including philosophers and theologians, Lonerganesque or other. Quinn explicitly reverses Lonergan studies without assembling much data, while Duffy suggests philosophers and theologians need to mind heuristic structures.

In her second objectification, Moloney notes that both climate change and the development of artificial intelligence are highly complex and require diverse scientific knowledge and technical expertise to manage. She adds that the situation generates debate, at times emotionally charged. This is also a positional element that merits further discussion. And since one of our primary focuses in this exercise is the meaning of probability in *Insight*, positional development requires readers of this work and *Method in Theology* on Dialectic, who privately or publically claim progress has been made in Lonergan studies in the last 60 plus years, to join the exercise. Duffy envisions a future when high school students will self-appreciate that the meaning of *probability* is a dialectic-genetic story and educators for progress will ask: "What might be a valuable statistics in 2020 for planning a more livable human life in 3020?"

Are there things in our individual objectifications that are counter-positional and merit reversal? Indeed there are, and identifying some of them is also a part of the procedure of asking "some basic questions, first, about others, but eventually, even about [myself]."[59] It is the naked high point in the turn from Dialectic to Foundations, the identification of something, someone better than was, and is, the reality.

Duffy moves into teacher mode while objectifying where he is from. He (I) had in mind the average reader, who might not be friends with the symbolization introduced by Quinn. It is possible that you, my reader, cannot auto-biographically position the meaning of *random* or *non-systematic*. I am too well aware of the menace of post-systematic philosophy and theology—the occasional employment of terms like *probability*, *random*, *emergent probability*, *exponential progress*, *systematic*, even *God*—under the

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[59] *Method in Theology*, 253.

influence of the academic disciplines.[60] But it is quite possible that you have no systematics of any of these terms, including *systematic*. So I slipped into teacher mode, which has no place in mature Dialectic.

On a similar note, I do not foresee Quinn's long footnote 35 on page 94 about intended readers having a place in mature Dialectic, in which there will be much that goes unsaid because those collaborating are not only "closer together" than the three of us, but also focused on communicating a very refined increment of progress to their "foundational neighbors." This is not so much counter-positional as it is Quinn's way of dealing with the meantime transitional period.

A shortcoming of the exercise surfaces in Quinn's distinction between "empirical" and "adverbial" probability, the latter having to do with questions of the type "Is it so?" "Will it be so?" and "Might it be done?" Neither Moloney nor Duffy worked with this distinction as such—although it is implicit in our concerns about the climate crisis—as we both focused on what Quinn calls "empirical probability." Quinn also has extensive comments on the emergently probable character of total process, which has me wondering if there was something "counter positional" in the decision to focus on *probability*.[61] Indeed, this has me wondering about how to do this exercise semi-decently. Perhaps it is *emergent probability*, not simply *probability*, which represents the fuller shakeup of the scientific community.[62]

A final comment about the (future) development of the entire exercise, indeed of all those attempting this exercise. We skipped over the tasks of *Completion* and *Comparison* for lack of competency. It is simply too soon. So I, for example, admitted that I am not positioned to pick out 50 or 100 good things in the researches, interpretations, histories, and events, statements,

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[60] See further the paragraph on "post-systematic literature," page 304 in *Method in Theology*. Also on my mind are the low and high rung "academic disciplines" mentioned in the second paragraph of page 3.

[61] This is a difficult point, one which begs the question "What are we doing, wanting to do, hoping to do, or pretending to do in this exercise?" Possibly helpful analogies for answering the question come from the history of the ongoing discovery of methods, for example, Einstein's nudge of the community of physicists. What is particularly troublesome is how to interpret and implement the discovery of a method (Dialectic) that sublates the positions of M. Planck and T. Kuhn.

[62] Note that Lonergan's "clarification by contrast" with four other views of world process (CWL 3, 151–161) prescinds from "tackling the larger issue" (612) of things.

movements having to do with the ongoing story of *probability*. Are Moloney's comments about "flaws in understanding, in one case of classical law, in the other case of statistical law" relevant to the assembly, interpretation, and evaluation?

My questions are question-begging: Who is this group of virtuous characters with the gall to do structured Dialectic for the love of God not an object? Dialecticians (and foundationers) are the "new" norm, but we are not them. So, how do we birth them? Quinn suggests something similar when he writes:

> I envisage (future) ongoing progress in finding and "organizing" "genera and species" of emergent empirical and adverbial probabilities, as well as emergent "genera and species" of empirical probabilities of emergent genera and species of empirical and adverbial probabilities.

That is quite a crazy sentence, even if your moving viewpoint made it as far as a self-revelation of "the heuristic significance of the notion of development" in preparation for "our statement of the integral heuristic structure that we have named metaphysics."[63]

There is a sentence in Quinn that did not jive with me, and I believe it merits further discussion, perhaps in another exercise. The sentence reads: "It is well known that world process includes vast ranges of (statistical) schemes of recurrence." I do not mean to quibble over words, but I wonder about the meaning of "well." Moloney recalls the canon of complete explanation, the high goal of empirical method, "the complete explanation of all phenomena." That surely would qualify as "well knowing." On a similar note—after all, *Insight* is just an invitation to work through 'apparently trifling problems' in route to the high-flying acrobatics of genetic method and methodical hermeneutics—"well" might mean luminous, evidently and self-evidently evident and self-evident speaking, listening, living, and longing for a "fusing into a single explanation," with the help of Chinese acrobats of meaning, each one doing his or her little part humbly and well.[64] Luminosity would mean that those "well

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[63] *Insight*, CWL 3, 484.

[64] This doing of little part humbly and well is intimated in Chapter 5, "Functional Specialties." "Without … what precisely they are doing" (137). Early on I had in mind to do a paragraph by paragraph, line by line analysis of how we three move in and out of proto-specialties in "vegetative" one-sided totalitarian ambitions. That would have been LOL humbling indeed.

knowing" the inclusion of statistical method in their philosophy (or theology) of total process—including future process—are self-assembling assemblers.

This lonely well-knowing, a "living, a developing, a growing, in which one element is gradually added to another and a new whole emerges,"[65] needs to be developed. So, besides the question about where is the story of probability theory going in the next 100 or 1,000 years, there is a further, related question regarding world view: Where are we to go in the next 100 or 1,000 years? Will our going be progressive? Will it be functional, proto-functional, and/or pre-functional? Might there be an exponential dimension to functioning? Moloney cites Bartlett on the shortcoming of not understanding the exponential function. But none of us went hog wild idealizing exponential functioners, elders with an integral perspective, mediated and protected by symbolisms, which ground the task of *Comparison*, which we conveniently skipped.

You or I can read the phrase about the immediate becoming mediated by a life of contemplation ontically, in terms of personal development; but we might also read it phyletically, the rescue of "well" as the destiny of history and her-story that is intimated in lonely heuristics, be they the chemistry of charity[66] or a stairway on and for earth and all its creatures as it is in heaven.[67] It is intimated in the lonely fact that consciousness "constitutes and reveals the basic psychological unity of the subject as subject." We three could have intimated a little better the fact that we do not know ourselves very well, that "our course is in the night" and that "we have to believe and trust, risk and dare."[68] How well do we know that world process includes statistical schemes of recurrence if our

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[65] CWL 6, 179.

[66] CWL 3, 763. See also the first paragraph on page 88 of "'MacIntyre and Lonergan' Revisited," *Journal of Macrodynamic Analysis*, vol. 12 (2020).

[67] See the staircase diagram on page 3 of Philip McShane, "Structuring the Reach towards the Future," an essay written for *The Third Peaceful Coexistence Colloquium*, Helsinki, Finland, June 13–14, 2019. The essay is available at: http://www.philipmcshane.org/ecornomics.

[68] CWL 4, 224.

objectifications gloss over or under the lonely subject-as-subject[69] in the madness and muddles of *Mo PoMo*?[70]

B. Cecilia Moloney

Lonergan writes about the lower level of the structure of dialectic that, following initial operations on assembled materials, a final objectification of horizon occurs when "the results of the foregoing process are themselves regarded as materials, when they are assembled, completed, compared, reduced, classified, selected, when positions and counter-positions are distinguished, when positions are developed and counter-positions are reversed."[71] This quote by Lonergan is dense, so let me start with the materials to be assembled, etc., and ask, What materials are to be assembled, etc.? What are the results of the foregoing process in this case?

I claim that the materials to be assembled, etc. are of two types. First are obviously the texts of the first two objectifications written by three authors above. Second is a set of materials and results from the process we have gone through so far. This set includes each of our evolving views and understandings about the process, its structure and purpose, as well as our developments of our horizons about the ideal process Lonergan intended.

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[69] "The subject as subject is reality in the sense that we live and die, love and hate, rejoice and suffer, desire and fear, wonder and dread, inquire and doubt." CWL 18, 315–16.

[70] *Mo PoMo* is the shorthand name we (my students and I) settled on for their last semester seminar "Modernity and Postmodernity," a course which I designed at taught 2011–2014. There were some "big name" philosophers in the list of assigned readings—Nietzsche, Lyotard, Foucault, Heidegger, Habermas—but the course was a disciplinary collage. We read feminist authors, Jane Jacob's *The Life and Death of Great American Cities*, Jorge Borges' "The Library of Babel" and "The Circular Ruins," and the first six chapters of Philip McShane's *Futurology Express*. In addition, we watched *Blade Runner* and *Wings of Desire*, as well as a documentary of the life of the choreographer Pina Bausch. What, you might ask, was the crazy meta-narrative on the mind of the professor? At one point I had in crazy mind to mess around with classical and statistical methods, the notion of schemes of recurrence, and probability of schemes.

[71] Lonergan, B.J.F. (1973). *Method in Theology*, 2nd ed. London: Darton Longman and Todd, p. 250. In the case of the present exercise, the "final objectification of horizon" that Lonergan refers to is this third objectification, following the first and second objectifications by multiple authors working in isolation.

I start with the results of the process, so far, as I have experienced it. Throughout this exercise I have struggled with reservations about the process, both in terms of the specific texts of this exercise and the overall purpose and form of the exercise.

When the invitation from James Duffy arrived on Aug 15, 2019, I was relieved that the text chosen for the exercise consisted of sections of *Insight* on probability and emergent probability,[72] and not as I feared a more difficult text such as that of a dialectic exercise initiated by Duffy earlier in 2019.[73] I felt I had some knowledge of probability that might enable me to undertake this exercise.[74] Nonetheless, it took me some time to accept Duffy's invitation, in part because I am temperamentally averse to conflict and have tended in the past to skip over Functional Specialty 4 in favour of the forward specialties that I am more drawn to. In the end, I wrote to James Duffy on August 27, 2019: "Thanks for your recent emails and invitation to participate in a dialectic exercise. While I still have some uncertainties about the process and purpose of the exercise, I would like to accept the invitation. I will sort out the process by doing it, I am sure (and hope!), and

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[72] Lonergan, B.J.F. (1992). *Insight: A Study of Human Understanding*. Eds. R.M. Doran and F.E. Crowe. Toronto: University of Toronto Press (CWL 3). Hereafter *Insight*.

[73] Duffy, J., Gillis, A., Henman, R., Quinn, T. and Zanardi, W., "Effective Dialectic Analysis" (May 2019), the first exercise in this volume.

[74] Because of word count limitations in the first and second objectifications, I omitted a paragraph with my credentials, as I called them at the time. Now I might call them my experience, which have given me a stand-point that I bring to this writing. Since it may be of interest to readers of my objectifications, I add this paragraph back now as a footnote. I am a scientist and work professionally as a professor of engineering. I studied probability in several undergraduate and graduate courses, both the problem-solving development of probability and the abstract measure-theoretic approach. I have taught courses in probability to electrical and computer engineering students; these courses, especially for undergraduate students, have a practical orientation and consequently are structured around problem-solving. My engineering research is in digital signal processing, and since observed signals always have a non-systematic component, my research necessarily employs statistical techniques based on probability theory. I have been a serious student of Lonergan for over 15 years, and via various attempts to apply Lonergan (as I have naively phrased it) for the professional practice of engineering, I have been forced to go beyond Lonergan's written words to their implications in my own life and work. To what extent I have been successful in this regard is a question requiring critical reflection.

also I think I may have something to contribute on the text sections you have selected on the meaning of probability and emergent probability."[75]

In October 2019 I started to review the texts in *Insight* in preparation for writing my first and second objectifications. Despite emails from James Duffy that were intended to be helpful and clarifying, and despite the example of the previous exercise,[76] I still found the exercise murky—both in structure and purpose. I enjoyed taking a deeper dive into the texts by Lonergan, and exploring other fundamental writings on probability. I had accepted the challenge of the exercise, thinking that I could learn more about dialectic by doing. Yet, with my review of the texts in *Insight*, I began to clarify my reservations.

A key reservation was that the exercise seemed to me to be rather artificial. Three of us were working in isolation on a broad and deep part of *Insight*. Even though the focus was meant to be probability and emergent probability, this ended up being an assembly of Chapters 1–4 of *Insight*, at a minimum, since probability comes in explicitly or implicitly throughout these four chapters. Some of the co-authors might not have worked in a scholarly way on these chapters in the past, and in this case, we might not have our own past objectifications to work from. Since the swath in *Insight* was broad, and the word count small for the first two objectifications, I thought it likely that the authors might focus on different aspects in the four chapters; in that case, we might have few common points to debate further in a scientific manner, or to assemble etc. in Lonergan's dialectical manner, in the third objectification.

That said, I could see where the exercise could be heading if we were doing it for longer, and could cycle round enough times. In that sense, it would be like scientific debate. But as it was, with the authors writing in isolation from one another, I had doubts that one round would produce much of interest. I can add here that this process is not how co-authored scientific papers are written.

When the collated version of the first two objectifications by the three authors arrived, I read the written texts with interest, as well as with some dismay. I wasn't sure what to do next, and I knew that resorting to the scientific peer-review that I am more familiar with would not be very helpful. My puzzling then focused on the purpose of the exercise, as this might give me a stronger clue about how to write a third objectification. I

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[75] In this part of my third objectification, I rely on my own notes and various emails to generate an assembly of materials.

[76] See the exercise mentioned in footnote #73.

certainly thought that each author wrote a text of their first and second objectifications that was coherent and worthwhile/interesting in itself. One concern was how others might read the three texts taken together. A reader not familiar with Lonergan's work, or even a reader who is a Lonergan scholar with little prior experience with probability, or one like myself who is not drawn to dialectic, might say, "What is all this about?" In making this statement, I was looking at the exercise from a pedagogical perspective, and for the potential of this exercise to demonstrate to others the merit of the approach. I wasn't sure that the exercise was a success from this perspective.

On the other hand, the aim might be primarily for the authors to learn how to engage in dialectic, which is also a worthwhile goal, and perhaps what is needed now. But in this case, I thought that we might want to re-assess the results before publishing the exercise, or to consider cycling round again and with more interaction between the authors. This continuing reservation with the exercise may stem from my scientific background,[77] as I see the flaw of this specific exercise on probability and emergent probability as starting from a topic that was too broad. For example, in my first two objectifications, I didn't even get past very basic probability, and certainly made no substantive comments on 'emergent probability' bar a few throw-away references to Lonergan on world process, but without the connection to emergent probability being discussed or even clearly made.

I realize that the goal of this exercise is not to emulate the scientific process as such, but rather to initiate a wider or generalized scientific process, one with direct speech.[78] But, the scientific model is still a useful one. In one of his emails James Duffy mentioned the scientific example of Maxwell nudging Faraday. In a similar way, I think we should be nudging each other on the same points. I realize Maxwell was nudging Faraday across the breadth of electromagnetics, and ultimately this is what is envisaged with respect to probability and emergent probability. But this cannot be done with any level of success for such a broad field in single short texts of 1800 words written in isolation. The exercise might have more success if we started with smaller sub-topics within the wider field, so that we are at least tackling the same mountain more or less in the same place.

In summary on the process, I found this exercise to be quite difficult and ultimately frustrating. Indeed had the topic of probability and

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[77] See footnote 74 above.

[78] *Method in Theology*, 267.

emergent probability not been an attractive one to me, I might not have continued to participate.

Now for some specific comments on the written texts of the first two objectifications. When I read the collated texts, I thought that the three authors all chose fairly different approaches in their first and second objectifications. But the texts that the three authors wrote, independently of one another, are not fundamentally in conflict. All three authors, in different ways, wrote about the everyday meaning of probability. They all addressed how the average person understands probabilities, via the impact of probabilities in their lives. At the same time, all three authors addressed the need for more precise, scientific understanding of probabilities and of the nonsystematic. All three authors also intended their discussion of probability to encompass the natural sciences and to extend to the human sciences, to education, and to many sectors of our society. The authors cited similar examples: Duffy and Moloney used climate change as an example, and Moloney and Quinn used artificial intelligence as an example. Duffy and Quinn extended their discussion explicitly to emergent probability, while Moloney only referred to emergent probability indirectly via brief references to Lonergan's perspectives on world process.

As a final note, I wrote my first two objectifications during the Canadian federal election campaign in fall 2019, when climate change was a key theme in the election conversation. I am finishing this third objectification at home in spring 2020 during the early days of social distancing during the global COVID-19 pandemic. Both climate change and the COVID-19 pandemic are emergent situations that need to be understood, in part, in terms of probabilities. Despite the importance of climate change, I think the probabilities of the pandemic are much more immediate for many people, as they are connected to such concrete and personal questions as: "Will I, or someone in my family, get the virus? "If so, will I survive?" "Will we ever get back to the way life used to be a few short months ago?" I would certainly pray that the answers to the first two questions would be "No" and "Yes," respectively, for all people who ask them. On the other hand, the third question might present an opportunity for dialectic, for developing an understanding of the past, as "something better than was the reality."[79] And perhaps our existential struggle with the probabilities of this year can encourage in many "the conscious occurrence

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[79] *Method in Theology*, p. 251.

of the intellectual events that make it possible to know what happens when probability is grasped."[80]

    C. Terrance Quinn

Preamble

I learned from my collaborators James and Cecilia; I was pushed to an improved heuristics of *probability*; and I was able to obtain a "glimpse or two" of our progress-oriented task.

    I began by working through our first and second objectifications[81]. Initially, this was interpretative but not "functional interpretation." What I first obtained was thin description. It seemed to me that our (pre-scientific) contributions to Part I were a kind of "eco-system" (with differences, commonalities and overlaps). One of the goals that took shape for me was to obtain some kind of (amateur) "evaluation and comparison." Some questions that emerged pointed well beyond my present reach. For instance, in Part I, how are we doing, individually; relative to each other; and, indeed, in history? Which aspects of our results contribute to our present purpose? Which elements might be redirected or perhaps "recycled"? Which might call for revision? It seemed helpful to attempt to identify "functional leans."[82]

    The prefix "proto-" is to be understood throughout. For instance, C12 stands for communications from (proto-) functional research to (proto-) functional interpretation, and so on.[83] Our efforts here are proto-F4. My admittedly *ad hoc* effort to take stock of our results from Part I are given in section Q0′.[84] Sections Q1′ and Q2′ are my "second first objectification" and

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[80] *Insight*, p. 82.

[81] In what follows, I call our first and second objectifications Part I, and our effort in the third objectification Part II.

[82] Part of Lonergan's 1969 discovery is that these are always present, even though so far that presence is mainly inadvertent and/or confused.

[83] I use McShane's notation: Fi for functional zones, and C(i, j), i, j = 1, 2, …, 9 for communications between functional zones. See, e.g., Philip McShane, *A Brief History of Tongue, From Big Bang to Coloured Wholes* (Vancouver: Axial Publishing, 1998), 108.

[84] Word limits remain in force. This called for brevity, which I found to be a challenging and helpful feature of the exercise. References to Part I are indicated by paragraph: (Duffy, ¶1), (Moloney, ¶1), (Quinn, ¶1), etc.

"second second objectification," respectively.[85] I end with a comment regarding the possibility of "further third objectification."

Q0′   Our first and second objectifications (in history)

All three of us are in a "mode of learning." But we are also trying to get beyond the contemporary ethos. This is evident in at least four ways: (1) We share a context of concern for global progress in the academy and society. (This includes wanting to solve the economics problem[86] and the ecological crisis.) (2) We affirm that there is "progress-potential" in Lonergan's results on probability (so far missed, ignored, or otherwise treated in old-fashioned philosophical debate remote to experience in empirical sciences). (3) Our efforts are *toward* functional dialectic[87] (although for now we are attempting proto-dialectic). (4) We take for granted that a growing control of meaning in sciences will be needed for "effective intervention in history."[88]

For "reversing counter-positions," all three of us advocate implementation of Lonergan's discoveries regarding the possibility of (balanced) empirical method. James makes use of engaging and accessible examples. Cecilia's discussion regards important contemporary issues. The rather generic "reversal strategies" indicated in my contribution to Part I need considerable elaboration and need also to engage with details of works of currently influential authors.

James describes instances of his understanding in "empirical probability." He also draws attention to another kind of probability but does not seem to address that in Part I.[89] I suspect that James has a solution in mind, which makes me think that word limits may account for that

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[85] The notation is to distinguish these from first and second objectifications D1, D2, M1, M2, Q1 and Q2, in Part I (Duffy; Moloney; and Quinn).

[86] It seems that there is consensus here that the economics discovered by Lonergan is what is needed. This reveals the need for further "assembly ..."

[87] The structure, discovered by Lonergan, is densely summarized in *Method* 250; CWL 14, 235.

[88] Bernard Lonergan, *Phenomenology and Logic: The Boston Lectures on Mathematical Logic and Existentialism*, ed. Philip J. McShane, vol. 18, Collected Works of Bernard Lonergan (Toronto: University of Toronto Press, 2001), 305-6.

[89] "In all probability, we won't meet the deadline." (Duffy, ¶2, p. 84).

omission.[90] In what I wrote, I referred to data but mainly only indirectly (my experience, e.g., in mathematical sciences). However, historically, similar experience is had by many. In what James and I wrote, there is a compatibility in that both of us affirm the need of "catching oneself" in the act of understanding (probability).

Under "first objectification," Cecilia seems to focus on "Lonergan's meaning."[91] Still, her comments helpfully reveal aspects of functional dialectic, about which I speak below. Note that, in (Quinn, ¶18, p. 101), I refer to "act." I intended for the context to imply, but did not write, 'proportionate.'

There are questions of audience and communication. Cecilia's reflections partly are an invitation to read and learn from Lonergan's work on probability. What James wrote may stand the best chance of helping contemporary readers concretely engage in the problem. Pretending that we were "three front-liners," I was mainly writing to James and Cecilia. However, I hope that my efforts also will be of interest to colleagues who are attempting similar exercises, at least by way of providing data.

Some functional leans: In addition to the main purpose of the exercise, James' work "has another face," in the sense that it will be of immediate value as C49. Cecilia's "first objectification" seems to lean toward F2. Implicitly, however, her comments there are also C42, for she helps reveal the need for interpretation of Lonergan's writings.[92] Cecilia explicitly calls for functional collaboration. That aspect of her work is, I think, in a C59 lean. In the first ten paragraphs of what I wrote, the dominant mode is description, in a C44 lean. In (Quinn, ¶11, p. 98), there is a misstep. The beginning of the paragraph is in an F4 lean. But, going on to write "I envisage (future) …" I inadvertently shifted into an F5 lean. The potential

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[90] Balanced treatment of all types of "probability" is needed. For centuries, confusions about different "kinds" of probability have been undermining work in philosophy, theology and sciences.

[91] "Lonergan employs …" (Moloney, ¶1); "Lonergan's interest ..." (Moloney, ¶2); "[Lonergan] develops…" (Moloney, ¶2); "What I think Lonergan meant …" (Moloney ¶2); "Thus Lonergan can offer the following summary …" (Moloney, ¶3). "Lonergan outlines…" (Moloney, ¶4).

[92] That will be a community-wide historical undertaking. Lonergan was a genius. Even if he had not gone on, his control of meaning, precision and nuanced reach in his early papers still remain well beyond the current ethos. See, e.g., Bernard Lonergan, *Archival Material: Early Papers on History*, vol. 25, *Collected Works of Bernard Lonergan*, eds. Robert M. Doran and John Dadosky (Toronto: University of Toronto Press, 2019).

effectiveness of the paragraph is undermined because "it is looking both ways."

Q1´   Second first objectification

I divide my comments into [a] and [b]. In [a], I provide a few points regarding our central topic, *probability*. These came up within this attempt at a third objectification. Statement [b] is by way of "supplementary first objectification." I draw attention to an issue that was implicit in Part I and explicit in Q0´. I single it out it because it is intended to help my collaborators know and evaluate "where I'm coming from" in Q0´.

 [a]   *Probability*

In scientific contexts, "counting" can be "non-trivial."[93]

I find that contexts shift in a major way when I include "the problem of metaphysical equivalence." Yes, in empirical probability, one counts [94] events or occurrences of "type X." "Type"? Philosophical debate aside, "classifications" have been and are being discovered, corrected, revised and advanced in ongoing scientific development. And classification includes classification of classification, and so also classification of discovery and verification of corresponding empirical probabilities of what has been classified.[95]

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[93] There are, for instance, probability measures computed and verified in quantum physics; in physical chemistry, we make use of Avogadro's number as well as variables in thermodynamics and statistical mechanics; ongoing progress in statistical methods is crucial in modern biochemistry, biology, environmental sciences and population dynamics. And so on. In other words, statistical methods and probability distributions have been emerging in the genetic-dialectics of historical development. Even apparently elementary questions pose challenging problems. What is a probability distribution for consecutive runs of Heads (or Tails) of various lengths? A well-known mathematical solution makes use of "recursive sequences" in a "space of sequences." The resulting distribution provides small but nevertheless positive fractions for arbitrarily long (finite) runs of Heads (or Tails). In experiments, however, runs of more than 10-12 Heads (or Tails) almost never occur. For *empirical* probabilities, then, there is a need for suitably structured cut-offs. What is "suitable"? That calls for further development and is determined by investigators.

[94] See note 93.

[95] I am referring to what is precisely (but densely) thematized in CWL 3, 457. Ongoing displacements are to be in a "comeabout" (CWL 3, 537). Not yet operative in the academy, needed heuristics are expressed in metagrams such as

I have found that still further displacements are possible in theological contexts, by adverting to "created participation in uncreated light," in experience. To hold all of this together, I find it necessary to make use of the precise and all-inclusive symbol $\{M\ (W_3)^{\theta\Phi T}\}^4$ invented by McShane.[96]

And so, the work of determining empirical probabilities for some X (including, certainly, when X is our own performance) is a particular case of progress in mature prayer-types. Again, when real X's vary "non-systematically," there is the possibility of growing in knowledge of prevailing trends in "relatively how many" and "relatively how often" real X's have been or generally are present, in us, in history.

[b]   *Supplementary first objectification*

As I mentioned in the Preamble, in Q0′ I partly looked to functional lean. The last paragraph of Q0′ makes this explicit.

Q2′ Second second objectification

A basic issue that came up in Q0′ is the task itself. In our "first objectifications," part of the task is to "distinguish positions" and "reverse counter-positions." (Although, once a standard model SM is established, I think that *fundamental* disagreement (e.g., regarding W1, W2, W3 …) will be more or less a thing of the past.) What I think warrants mention here is that the two aspects of the first objectification just mentioned seem to me to be "two faces of the same coin," or rather, "two *faceres* of the same going."[97] Perhaps this is too obvious to need saying: As needed and possible, dialectic will include: "orienting" and "re-orienting" oneself and others, *relative to the current front-line*.

In the autumn of 2018, I had the pleasure of attending a performance of *Lakmé*[98]. As I finish these few paragraphs, I am listening to the Flower Duet. And so, I am also thinking of the "Flowering of Dialectic." I take the liberty of adding two lines to the libretto:

W1, W2, W3 … (See Philip McShane, *Prehumous* 2, "Metagrams and Metaphysics," available at: http://www.philipmcshane.org/prehumous.)

[96] Philip McShane, "*LO and Behold* 11: Assembling $\{M\ (W_3)^{\theta\Phi T}\}^4$," available at: http://www.philipmcshane.org/lo-and-behold.

[97] I am thinking of the etymology of the word 'face': *facere*, "to make" (from PIE root *dhe* – "to set, put").

[98] The Flower Duet is from *Lakmé*, an opera in three acts by Léo Delibes, to a French libretto by Edmond Gondinet and Philippe Gille. The score was written in 1881–1882. The opera was first performed on 14th of April 1883, by the *Opéra-Comique* at the (second) Salle Favart in Paris.

Let us descend together!

…

Under the thick dome where the white jasmine

Ah! calling us

Together!

[Ah! Calling us

Onward!]. [99]

Dialectic will be a "togethering, for onwarding." But what are the increments dSM to be chosen for our onwarding? One will need an operative grasp of the up-to-date standard model (which will include increasingly nuanced renderings of W1, W2…[100]). The "six tasks" described in *Method* 250 will be major zones of scientific inquiry[101]. And, within eight functional groupings, functional dialectic will be a "MEGA Science," a "Method of Evaluation, for Group Acceleration."

What does all this have to do with *probability*?' I would not presume to know what the genius Lonergan meant in the last few paragraphs at the end of Chapter 5 of *Insight*. But for us, for history, it seems to me that the "concrete intelligibility," as such, of emergent probability *never will be reached*.[102] The "process is in process." And emergent and shifting events and occurrences are in numbers and time spans that defy imagination. But functional dialectic will be a statistically effective part of our part in the process-in-process that is our whole bloody and Blooded story.[103]

A note regarding possibly continuing the third objectification: Except for being asked to share our first and second objectifications, we were asked to not be in touch with each other while working on this exercise. At this point,

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[99] Third stanza of the translated duet (Aaron Green, tr. "Flower Duet Lyrics and Text Translation," "*Sous le dôme épais*," "Aria from *Lakmé*," April 1, 2018. https://www.liveabout.com/flower-duet-lyrics-and-text-translation-724326. Accessed November 27, 2019. I added the words in square brackets.

[100] For instance, there will be luminous and increasingly nuanced heuristics for the biochemistry of "types" (see Q1′ [a]) of aggreformic entities.

[101] There is, for instance, Philip McShane's breakthrough in heuristics of "comparison" (Philip McShane, *Method in Theology 101 A. D. 9011, The Road to Religious Reality* (Vancouver: Axial Publishing, 2012). For preliminary efforts in evaluation of McShane's discovery, see the first exercise in this volume.

[102] Lonergan, CWL 3, 195.

[103] Colossians 1:20.

however, there is nothing to prevent the three of us from continuing our exchange regarding *probability* or, indeed, entering into new and similar exercises about other materials assembled. This could lead to further mutual growth. Eventually, one or more of us might get to something that we think is worth communicating (forward), something that would contribute to improved foundations. This raises further and "furthering" issues.

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