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THE BASIC PRICE SPREAD RATIO

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This essay endeavours to follow my reading of the argument in Bernard Lonergan’s quite brief discussion of the above topic, to be found in *Macroeconomic Dynamics: An Essay in Circulation Analysis*, Collected Works of Bernard Lonergan 15 (Toronto: Toronto University Press, 1999) (hereafter *CWL 15*), as §28 (pages 156-162).[1]

An immediate difficulty must be faced. Section 28 occurs late in Lonergan’s development of his argument, and must therefore build on concepts and ‘theorems’ that he has introduced in earlier sections. I can indicate when these are used, but it would be unreasonable to expect their conclusions to be justified again here.[2]

Apart from minor changes in notation, etc., and some greater detail in the use of mathematical arguments, there is little that is novel in what is offered. It merely reflects what I found helpful, and the augmentations I needed, in my own attempts to grasp Lonergan’s arguments. It is tendered here in the hope that some other readers may find it helpful, and where

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[1] A related previous discussion was given in *For a New Political Economy*, Collected Works of Bernard Lonergan 21 (Toronto University Press, 1991) (hereafter *CWL 21*), as §15, on page 301. It concerned the Basic Price Spread difference itself ($P' - \pi'$), rather than the ratio of these two. It would take us too far afield here to analyse in detail why it failed to deliver the results Lonergan sought, and so was dropped in favour of the present approach.

[2] This essay is in effect just one chapter in a longer and more ambitious project to integrate/paraphrase the whole of *CWL 15* (and the *For a New Political Economy* essay in *CWL 21*.) Points used in the present chapter/essay will then have been justified in earlier parts of this much larger text.

I have got it wrong it may perhaps spark off a debate.

The essential point made by Lonergan is that the cyclical variations in the ratio under discussion are treated as signals of what is happening in the economy. It is because these in practice are often misinterpreted (primarily because of misleading underlying theory rather than as a result of malevolent greed) that the ongoing ‘pure cycle’ becomes corrupted into the boom and bust of the ‘trade cycle’.

Notational Conventions.

Following *CWL 15*, flow variables (so much every so often) are indicated by upper-case letters. Any exceptions will be noted where they occur. The related quantity variables, if needed, will be indicated by the corresponding lower-case letters.

All fractional variables and index numbers that are newly introduced in this section are indicated by lower-case Greek letters. They will either be direct transpositions of Lonergan’s Roman lettering or will have some convenient mnemonic value. For variables carried over from earlier sections of Lonergan’s text it would cause unnecessary confusion to change these, so they are kept more or less as given in *CWL 15*. Again, any slight differences will be pointed out when they occur.

The Diagram.

A great deal of Lonergan’s analysis is based around his famous Diagram, of which there are a number of different versions. I have had the temerity to give my own version, as *Figure* 1 (below). The following are its essential differences from Lonergan’s presentation: -

(*i*) It follows Philip McShane’s rotation of the presentation so that the two Surplus functions are on top and the two Basic ones are below.[3]

(*ii*) The Basic Supply and Demand areas are switched, so that the direction flow in both stages is the same (left-right). To avoid the two ‘crossover’ components having to be drawn on

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[3] Philip McShane, *Economics for Everyone* (Halifax: Axial Press, 1998), 92.

the diagonals, and so overlapping confusingly with the Redistributive area, this change necessitated some diversion of the arrows.

(*iii*) To meet the difficulty arising from *CWL 15*'s duplicated use of fractions, *s*' and *s*", two new ones, *e*' and *e*", have been introduced. Notice that the positioning of these four flows into ***Rf*** means that we depart from the *CWL 15* equations in that for each stage we in fact only have: $i + c = 1$.[4]

The Diagram.

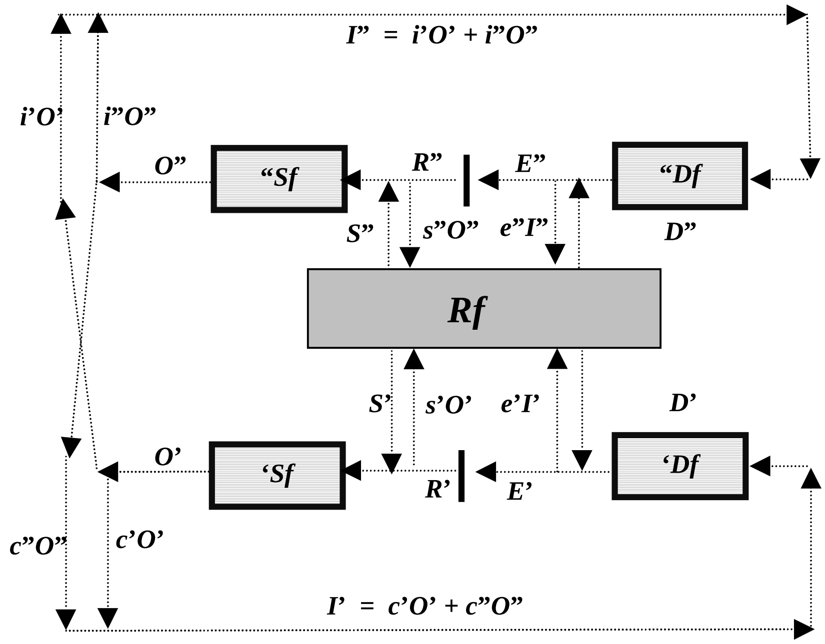

Figure 1.

The Analysis.

Lonergan begins by saying that there is a sense in which the portion[5] of Basic outlay [= ***c'O'***] that moves to Basic

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[4] This equation introduces another convenient notational abbreviation. Where variables that would normally be accented (to distinguish Basic and Surplus) are used in unaccented form, the absence indicates that what is said applies equally to both cases.

[5] Lonergan repeatedly uses the word 'fraction' to mean a part of something, so that in the case in question this would be an amount of money per interval. It seems preferable to substitute the word 'portion' and keep the term 'fraction' as meaning a pure number, the ratio of that part to

income is the 'cost' of Basic production.[6]

This certainly seems counterintuitive. In truth the economy is only 'for' the Basic stage, aimed at an emergent material and cultural standard of living (***SOL***). As this is its 'end', the Surplus stage is only a 'means'. It is of course not just a means to ensure the existence of a Basic flow, but essentially intends the latter's further growth. It behaves as a kind of bonus over and above the mere persistence of that Basic flow. But it is not pure bonus. At any particular moment some of it is required, in the form of depreciation (which we have written as ***Dep***), (*i.e.*, the demand for maintenance and replacement). The rest, what Lonergan refers to as 'net fixed investment' (***NFI***), is the true bonus.[7]

Since we are assuming at this stage in Lonergan's argument that the 'continuity condition' [$\boldsymbol{D' - e'I' = 0 = D''- e''I''}$] applies, then $\boldsymbol{I''}$ is keeping pace with $\boldsymbol{E''}$. If we therefore project the same proportional breakdown that we have in $\boldsymbol{E''}$ [*i.e.*, $\boldsymbol{NFI : Dep}$] backwards onto $\boldsymbol{I''}$ we get a partition of $\boldsymbol{I''}$ into what he calls pure surplus income (***PSI***)[8] and ordinary surplus income (***OSI***). In symbols: -

$$NFI : Dep = PSI : OSI \, .$$

Of course $\boldsymbol{I''}$ has another partition as well, based on its sourcing in either Surplus or Basic outlays. [*i.e.*, $\boldsymbol{c''O'' : c'O'}$.] We have no good reason for assuming that these two partitions would be the same, nor even merely in the same proportion. We shall have to return to this important point later.

PSI is the income equivalent of the 'bonus' discussed above. As a result of this analogy Lonergan can use it as his

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the whole.

[6] Recall that for us $i'' = 1 - c''$ and $i' = 1 - c'$.

[7] This terminology had all been set out by Lonergan in his previous section (§27)

[8] It almost goes without saying that Lonergan's use of the term 'surplus' for the circular flow made up of all the accelerator stages is immensely irritating. After all, the term 'pure' in "pure surplus income" should really just be the (ordinary usage) term 'surplus', meaning 'excess'. I think this whole matter needs amending in the tradition, but it would introduce too many distractions to attempt it here.

definition of a macroeconomic (= functional) notion of *profit*.

This is not of course an accountant's simplified view of 'profits', as the excess of receipts over outlays. The bulk of the latter are included by us as outlays, as being in a sense the 'wages' paid to managements or owners.

Lonergan does not say this, but symmetry would lead one to assert that there are two kinds of 'profit', relating to the two stages. Let us redesignate the profit defined above as *Surplus Profit*. In the Basic case the total income, ***I'***, could be similarly partitioned into pure Basic income and ordinary Basic income. The latter would act to maintain the existing ***SOL***. The former would be what enabled its growth, and could reasonably be referred to as *Basic Profit*.

In normal usage the obverse of 'profit' is 'cost'. But Lonergan makes an unexpected change. It would have seemed that the Surplus Cost might most simply have been defined as the income equivalent of Depreciation, and Basic Cost as the income equivalent of simple maintenance of an existing *SOL*.

Recall the point made earlier that there is no good reason for assuming that these are the same partitions of the two total incomes as the partitions that are made on the basis of their sources in Outlays.

Despite this *caveat*, Lonergan opts instead to use just such an outlay-based division to define his term 'Basic Cost' as being precisely ***c'O'***. [And 'Surplus Cost' as being ***c"O"***.] This is a little surprising. He may, of course, define his terms as he wishes, but since he has already fixed the notion of 'profit' this now means that 'profit' and 'cost' in his macroeconomic sense are no longer obverse terms. The remainder of Basic outlay, ***c'O'*** is not at all the same as 'profit' in the sense in which he has defined it.

It will be important to keep in mind that our intuitive notion of what 'basic costs' should(?) mean will not follow the theoretical definitions. We shall have to be wary.

In fact, of course, attempts at descriptive justification are not important. We are proceeding by developing our own definitions, some of them 'implicit'.[9] Ultimately the

[9] This is where terms define a relation and the relation defines the terms, and this alone is treated as being sufficient, there being no additional

justification of such invention is that it ‘works’, in that it gives us a powerful explanatory context.

For brevity of expression the term “Basic costs” will hereafter mean ***c’O’***. The fraction of Basic outlay that moves to Basic income is an *index* of the ‘cost’ of Basic production.

Recall that total Basic income is given by

$$I' = c'O' + c''O'' \qquad [4]^{10}$$

Lonergan has already discussed how ***O’*** and ***O”*** are functions not of the quantities ***Q’*** and ***Q”*** currently being sold at the two final markets, but of the corresponding quantities that are in production, which may be more or less than these, and which are designated as $\boldsymbol{\alpha' Q'}$ and $\boldsymbol{\alpha'' Q''}$.[11] (He referred to these α values as acceleration coefficients. We shall see another way of viewing them later).

It follows that each ***O*** is some price index multiplied by the corresponding $\boldsymbol{\alpha Q}$. This will give an equation of the form $\boldsymbol{cO} = \boldsymbol{\pi\alpha Q}$ for each of the two stages.

We will therefore define two *cost price indices* by

$$\pi' = \frac{c'O'}{\alpha' Q'} \text{ and } \pi'' = \frac{c''O''}{\alpha'' Q''} \qquad [41, 42]^{12}$$

This means that

$$I' = \pi'\alpha' Q' + \pi''\alpha'' Q''$$

When $\boldsymbol{D' - e'I' = 0}$, which is a general condition of circuit balance, we have $\boldsymbol{E' = I'}$. In addition,

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requirement imposed that would necessitate one delineating a meaning for the terms descriptively.

[10] I will use the equation numbering in *CWL15*. In the case of the present equation [4], however, note that as presented here it has no equivalent of Lonergan’s ***sO*** term. This most adequately meets the point made in footnote 57 (page 49) of *CWL15*. It is already clearly implied in our version of the diagram in Figure 1 above.

[11] E.g., *CWL 15*, p. 112-113.

[12] π replaces Lonergan’s *p*. We should also replace the later *P* (in *P*’) but this might cause confusion.

$$\boldsymbol{E' = P'Q'} \qquad [28]$$

where ***P'*** is the Basic selling price index.[13]

Putting this all together we get: -

$$\boldsymbol{P'Q' = \pi'\alpha'Q' + \pi''\alpha''Q''}$$

Now divide through by $\pi'Q'$: -

$$\frac{P'}{\pi'} = \alpha' + \alpha''\frac{\pi''Q''}{\pi'Q'}$$

Let us introduce two new fractional variables and terms.

$$\boldsymbol{\rho} = \frac{\pi''Q''}{\pi'Q'}$$

will be called the *Surplus to Basic Ratio*. [A]

$$\boldsymbol{\beta} = \frac{P'}{\pi'}$$

will be called the *Basic Price Spread Ratio*.[14] [B]

So now we have:

$$\beta = \alpha' + \alpha''\rho \qquad [45]$$

Before proceeding, it would seem worthwhile to pause for a moment to try to 'get our minds around' these two new concepts, $\boldsymbol{\rho}$ and $\boldsymbol{\beta}$.

By its definition,

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[13] Notice that in this instance, despite being upper case (and not being a Greek letter), ***P'*** is not a rate.

[14] ρ (rho) replaces for Lonergan's *R* which was altogether too problematic, suggesting the word 'receipts'. Recall that in this essay we are considering the Basic Price Spread Ratio. As already mentioned in footnote 1, Lonergan's earlier (unsuccessful) treatment was of the cyclic properties of the Basic Price Spread itself, $P' - \pi'$.

$$\boldsymbol{\beta} = \frac{\text{price index for Basic goods sold}}{\text{cost - price index of current Basic production}}$$

By [41] it can also be written: -

$$\boldsymbol{\beta} = \frac{P'\alpha'Q'}{c'O'} =$$

$$\frac{\text{current Basic PRODUCTION but priced at present Basic SELLING prices}}{\text{Basic costs}}$$

$$= \frac{\text{PROJECTED value of current Basic production}}{\text{Basic costs}}$$

The source of the Basic Price Spread is the difference between[15] Basic Receipts, ***R'***, and Basic-Outlay-sourced Basic Income. [More will be said on this below.]

$\boldsymbol{\rho}$, on the other hand, can be expressed by: $\boldsymbol{\rho}$ =

$$\frac{\text{present Surplus 'finishing' but priced at present Surplus COST prices}}{\text{present Basic 'finishing' but priced at present Basic COST prices}}$$

$$= \frac{\text{REPLACEMENT cost of Surplus goods sold}}{\text{REPLACEMENT costs of Basic goods sold}}$$

The second form shows that $\boldsymbol{\rho}$ is the ratio of <u>replacement</u> costs (*i.e.* it depends on $\frac{\pi''}{\pi'}$ rather than on $\frac{P''}{P'}$). But these two price ratios will be approximately the same.

This means that the essential variability of $\boldsymbol{\rho}$ is with the ratio $\frac{E''}{E'}$ [16], and so with $\frac{I''}{I'}$. This makes it correspond more closely to our intuitive expectations of what a 'surplus to basic

[15] Lonergan's use of the expression 'difference between' can be confusing. It does not always mean, as it would for a mathematician, the <u>result</u> of a subtraction, but seems to be just a synonym for 'distinction between' or 'non-equality of'.

[16] Since $\frac{\pi''Q''}{\pi'Q'}$ is approximately the same as $\frac{P''Q''}{P'Q'} = \frac{E''}{E'}$.

ratio' would be.

Because $\boldsymbol{\beta} = \alpha' + \alpha''\boldsymbol{\rho}$, the influence of $\boldsymbol{\rho}$ on the Basic Price Spread is mathematically clear. Since $\boldsymbol{\alpha''}$ is positive, increasing $\boldsymbol{\rho}$ will mean increasing $\boldsymbol{\beta}$.

But it is also easy enough to understand.[17] The greater the value of $\frac{I''}{I'}$, which we have just seen to be effectively the same as $\boldsymbol{\rho}$, the greater ***I"*** will be as a fraction of total Basic Income ***I* = *I"* + *I'***. This will tend to feed through to mean a greater contribution from Surplus outlay to the Basic stage, and so in turn a lower such contribution from Basic outlay. But this latter is just another way of saying a lower level of Basic Cost. Finally, since Basic Cost is the denominator of its alternative fractional expression, $\boldsymbol{\beta}$ will increase as well.

Let us now investigate $\boldsymbol{\rho}$ in greater detail. It can be written as $\left(\frac{\pi''}{\pi'}\right)\left(\frac{Q''}{Q'}\right)$.

For any variable *X* the <u>proportional rate of change</u> of *X* is $\frac{dX}{X}$.[18]

A notational convention.

Hereafter we shall write $\widehat{X}$ to indicate this <u>proportional rate of change</u> , [*i.e.* $\frac{dX}{X}$].

[This is traditionally read as "*X* hat".]

It is fact (based on fairly straightforward calculus considerations[19]) that if both ***Q*** values are positive then the <u>rate</u>

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[17] The formalism of mathematics is not a substitute for understanding. Indeed, it is too frequently a mask for its absence. You can switch off your head and just 'let your mathematical fingers do the walking'!

[18] An example would be the traditional notion of a growth rate (of GDP, for instance).

[19] <u>Proof</u>. For brevity, let us write *d* for $\frac{d}{dt}$.

$d\left(\frac{A}{B}\right) = \frac{BdA - AdB}{B^2} = \frac{A}{B}\left(\frac{dA}{A} - \frac{dB}{B}\right)$, by using the 'Quotient Rule'.

<u>of growth</u> of the ratio $\frac{Q''}{Q'}$

will be respectively exactly as	$> 0,$	$= 0$	< 0
	$\widehat{Q}'' > \widehat{Q}'$	$\widehat{Q}'' = \widehat{Q}'$	$\widehat{Q}'' < \widehat{Q}'$

In $\boldsymbol{\rho}$, however, there is another extra multiplier, $\frac{\pi''}{\pi'}$ of the ratio $\frac{Q''}{Q'}$.

Normally we would expect $\frac{\pi''}{\pi'}$ to be close to unit value, or at least fairly constant, since cost prices, *e.g.* wage rates, in the Surplus and Basic stages are determined by more or less the same considerations. If there is a difference it might be expected that $\boldsymbol{\pi''}$ will be higher than $\boldsymbol{\pi'}$ (*e.g.* wage rates in hi-tech industries, where a larger than average proportion of the production might reasonably be expected to be Surplus, may be higher than the general rates in the economy). But even then one would expect the ratio to be relatively constant. This implies that the <u>rate of growth</u> of $\boldsymbol{\rho}$ will be of the same sign as that of $\frac{Q''}{Q'}$. [20]

Since the conditions $\widehat{Q}'' > \widehat{Q}'$, $\widehat{Q}'' = \widehat{Q}'$ or $\widehat{Q}'' < \widehat{Q}'$ are

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With A and B positive, this means $d\left(\frac{A}{B}\right)$ and $\frac{dA}{A} - \frac{dB}{B} = \widehat{A} - \widehat{B}$ have the same sign. [Proved]

[20] If $X = kA$, where k is a constant, then $\widehat{X} = \widehat{A}$. [*i.e.* <u>not</u> $k\widehat{A}$, as would be the case with an ordinary derivative.]

<u>Proofs.</u>

[Each of the proofs uses the fact that k is a constant, so that $\frac{dk}{dt} = 0$].

(*i*) $X = kA \Rightarrow \ln X = \ln A + \ln k$

Differentiate *w.r.t.* time: $\frac{1}{X}\frac{dX}{dt} = \frac{1}{A}\frac{dA}{dt}$ <u>QED</u>.

Alternatively, (*ii*) using the product rule,

$$\frac{dX}{dt} = k\frac{dA}{dt} \Rightarrow \frac{1}{X}\frac{dX}{dt} = \frac{1}{\cancel{k}A}\cancel{k}\frac{dA}{dt} = \frac{1}{A}\frac{dA}{dt}$$ <u>QED</u>.

respectively those for Surplus Expansion, Proportionate Expansion and Basic Expansion,[21] these three are therefore correlated exactly with the cases $\boldsymbol{d\rho} > \mathbf{0}$, $\boldsymbol{d\rho} = \mathbf{0}$ and $\boldsymbol{d\rho} < \mathbf{0}$ respectively.

This can be conveniently assembled into a Table.

$\widehat{Q}'' > \widehat{Q}'$	$\widehat{Q}'' = \widehat{Q}'$	$\widehat{Q}'' < \widehat{Q}'$
Surplus Expansion	Proportionate Expansion	Basic Expansion
$\boldsymbol{d\rho} > \mathbf{0}$	$\boldsymbol{d\rho} = \mathbf{0}$	$\boldsymbol{d\rho} < \mathbf{0}$

Figure 2.

The next section in Lonergan's text was in his earlier notes, but was marked for exclusion from the 1978 version. This was not because it was incorrect, nor even uninteresting, but because it is difficult for most people and the central thrust of the argument can be sustained without it. Lonergan probably realised that for his long-suffering students the cake was just not worth the candle, and so decided to leave it out. Having investigated the effect of $\boldsymbol{\rho}$ on the values of the Basic Price Spread Ratio, he wished to discuss the effects of the two acceleration coefficients, the α values. The reader may treat my discussion (with a side bar and between the horizontal lines) as a long parenthesis, and simply skip it.

Let us begin with the accelerator equation $\boldsymbol{q} = \alpha \boldsymbol{Q}$.[22]

Differentiate this: - $d\boldsymbol{q} = \alpha d\boldsymbol{Q} + \boldsymbol{Q} d\alpha$

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[21] Note that these are the conditions for Surplus Expansion, *etc*. (as in Lonergan's original version given in *CWL21*). It is not in fact the confused amendment [*e.g.* $|\widehat{Q}''| > |\widehat{Q}'|$] suggested in *CWL15* . The addition of the absolute-value signs would make the working of the present analysis quite impossible, and is in any case even inconsistent with Lonergan's own statements (and usage).

[22] Recall that when variables for which one would normally expect accenting are written unaccented this is to be taken as implying that the analysis applies equally well to the Basic and the Surplus cases. Notice also that the use of a lower case ***q*** here breaks our normal notational convention. It still represents a rate, but the one that applies at a different moment in the process.

i.e. $\boldsymbol{Q}d\alpha = d\boldsymbol{q} - \alpha d\boldsymbol{Q}$

$$d\alpha = \frac{1}{Q}dq - \alpha\frac{dQ}{Q} = \frac{\alpha}{q}dq - \alpha\frac{dQ}{Q}$$

$$= \alpha\left(\frac{dq}{q} - \frac{dQ}{Q}\right)$$

$$d\alpha = \alpha(\widehat{q} - \widehat{Q}) \qquad [46]$$

Dividing by α gives: - $\widehat{a} = \widehat{q} - \widehat{Q}$ [C][23]

An analogy.

Before proceeding it may help to consider a better-known example of such an equation.

If nominal (quoted) interest rates on savings are *r* [*r* % expressed as a decimal] and inflation is running at ϕ [again a decimal] then we say the 'real' interest rate is $r - \phi$.

There is a true sense in which the interest rate *r* paid on savings *s* is exactly what we mean by $\widehat{s}$. Similarly, the inflation rate ϕ could be expressed, in terms of price *p*, as $\dot{p}$. The 'real' interest rate will then be $\widehat{s}$ - $\widehat{p}$.

Let us say that the real interest rate measures the growth of something we could call *worth*, *w* (or better, perhaps, '*purchasing power*')

Then we have: $\widehat{w} = \widehat{s} - \widehat{p}$ [24]

The analogy between this [$\widehat{w} = \widehat{s} - \widehat{p}$] and (my) equation [C] above is obvious.

q represents the quantity in production, and ***Q*** the quantity sold in the same interval.

Thus $\widehat{\alpha}$ could be described as the 'real' growth rate of the laying down of production (*i.e.* net of 'losses' to sales!) α itself might therefore be described as the 'production power' of the stage in question.

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[23] This result can reached more quickly by just applying to the ratio α = *q*/*Q* a general growth-rate Theorem that if X = $\frac{A}{B}$ then $\widehat{X} = \widehat{A} - \widehat{B}$.

[24] Notice of course that would be quite <u>meaningless</u> to move from this to *w* = *s* – *p*. The 'hat' is not an derivative that could, as it were, be integrated away.

Back to the analysis.

Let us now return to analysing the equation

$d\boldsymbol{\alpha} = \boldsymbol{\alpha}(\widehat{q} - \widehat{Q})$.[25]

Since we always have $\boldsymbol{\alpha} > 0$, the equation tells us that $d\boldsymbol{\alpha} > 0$ will coincide with $\widehat{q} > \widehat{Q}$. Let us assume that the phase represented by $\boldsymbol{\alpha}$ is an expansion.[26] This means that we have $\boldsymbol{\alpha} > 1$

The next section of Lonergan's text is made excessively difficult to read because of his (recurring) use of such long winded expressions as 'the rate of current production of … quantities … in proportion to its size'. The latter just means the growth-rate of current production, or symbolically, $\widehat{q}$.

Imagine that one was attempting to hold $\boldsymbol{\alpha}$ constant, at some value greater than 1. So one would be trying to maintain $\boldsymbol{q}$ as some fixed multiple (> 1) of $\boldsymbol{Q}$. This would imply that $d\boldsymbol{\alpha} = 0$ and so $\widehat{q} = \widehat{Q}$. But as soon as each component in $\boldsymbol{q}$ reached the final market, it would become part of the new value of the sales, $\boldsymbol{Q}$, a part that, on our assumption, is greater than the equivalent had just been. This means that $\boldsymbol{Q}$ would undergo an acceleration.[27] This would make $\widehat{Q}$ exceed $\widehat{q}$, so that $d\boldsymbol{\alpha}$ would become negative. $\boldsymbol{\alpha}$ would fall in value. So the acceleration coefficients are, as Lonergan remarks, 'magnificently unstable'.

The only way to ensure the continuance of a high value of $\boldsymbol{\alpha}$ would be to maintain $\widehat{q}$ at a constant positive value. Lonergan has already shown that $\widehat{q}$ = constant means that $\boldsymbol{q}$

[25] We are still within the 'parenthesis'.

[26] *i.e.* if we are thinking of $\boldsymbol{\alpha}'$ it is a Basic expansion, and if $\boldsymbol{\alpha}''$ a Surplus one.

[27] There are two points to note here. First, if there was only <u>one</u> good, the transition would be sudden and discontinuous, so that it would hardly be called an acceleration in any ordinary sense of the word. But Q is an aggregate of many smaller parts, and these will be sold at slightly different times. This means that the aggregate value will in all likelihood change more continuously.

The second point is that the word 'acceleration' refers to dq [or more adequately, dq/dt , and recalling that q is already a flow] , not to $\widehat{q}$.

itself has to be growing in geometrical proportion[28], a near miraculous situation that could not be long sustained.

The outcome of all this is that α rises to a maximum, and then stays there for as long as $\hat{q}$ can maintain its constant value. But this cannot be for very long. Eventually, therefore, $\hat{q}$ will have to fall, (so that $\boldsymbol{q}$ itself will begin to rise ever less rapidly), and then the value of α will begin to drop.[29] In accordance with equation [45] this will in turn lead to a drop in the Basic Price Spread Ratio.

<u>In any expansion</u> the lag between quantities sold, $\boldsymbol{Q}$, and quantities in production, q, means that we will have $\boldsymbol{\alpha > 1}$. In a controlled economy the $\boldsymbol{\alpha}$ values might conceivably be held at their 'theoretical' values, but in 'free' economies there can be no such restriction. Additional amplification effects (or their opposite – 'de-amplification'?) will be possible because of speculation, bull or bear.

Consider again: - $\boldsymbol{\beta = \alpha' + \alpha''\rho}$

This can be differentiated: $\boldsymbol{-d\beta = d\alpha' + \rho\, d\alpha'' + \alpha'' d\rho}$

Lonergan refers to the 'cyclic' factors in this. By this he presumably means the factors affected by the current phase of the cycle, which are $\boldsymbol{\rho}$ and $\boldsymbol{d\rho}$.

In all three cases, $\boldsymbol{\rho}$ is just a fraction, (and probably very small, since in general $\boldsymbol{Q''}$ is much smaller than $\boldsymbol{Q'}$), and it will remain one. This ensures that $\boldsymbol{d\rho}$, as a change in such a number, is itself also a fraction.

As long as we have expansion at all (of any of the three kinds), $\boldsymbol{\alpha''}$ and $\boldsymbol{\alpha'}$ will both be greater than unity. It will perhaps be easiest to assure oneself of this by viewing the sample graphs drawn by Lonergan, and to be found in *CWL15, 122 and 124*.

Let us now consider the modifying effects of the $\boldsymbol{\alpha}$ values, and in particular of the $\boldsymbol{d\alpha}$ ones,[30] on these general comments,

[28] See at *CWL 15*, p. 120. The mathematical background to this assertion is given in the <u>Appendix</u>.

[29] This is the end of the 'parenthesis'.

[30] Recall that there is no necessary connection between the α and $d\alpha$ values. A comparison would be the distance x travelled by a car and the speed dx/dt at which it was then travelling. One can proceed at more or less

for each of these phases in turn.

Surplus Expansion. [$\hat{Q}'' > \hat{Q}'$]

We have seen that this means $d\rho > 0$ and so ρ itself will be increasing. Since α' and α'' are both greater than 1 this means in turn that the Basic Price Spread, β , will also be increasing rapidly.

For recall that: - $d\beta = d\alpha' + \rho\, d\alpha'' + \alpha'' d\rho$

Every part of this is positive, and so $d\beta$ is resoundingly positive.

$\rho = \dfrac{P'}{\pi'}$ could, in strict mathematics, be increasing either because P' was increasing or because π' was decreasing. But especially in a situation of rising production it is extremely unlikely that π' would be decreasing. [For instance, in an expansion acceptance of pay reductions by workers will be at its least likely.] In fact, therefore, it will mean that P' is rising. Such rising selling prices may call out speculators, who will always want to 'go while the going is good'. Such speculative money will flow into even more production, so further augmenting α' and α'' . α'' will mount to reach its maximum and stay there (with $d\alpha'' = 0$) . α', on the other hand, will mount initially but then contract (*i.e.* $d\alpha'$ will fall back to negative values.)[31]

In the initial step the increases will swing back again through $\beta = \alpha' + \alpha''\rho$ to expand the Price Spread even more. And this will repeat in a positive feedback loop.

In phase 2*b* the positive $d\rho$ can mitigate the effect of the negative $d\alpha'$ so than $d\beta$ can stay positive for longer than might

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any speed (subject to common sense and police control!) at any particular point.

[31] Compare sub-phases 2*a* and 2*b* in Lonergan's illustrative graphs in *CWL15* (particularly page 123). This is one of the points referred to in the Preamble, where practicality means that we must simply proceed on the basis of his earlier results, since any attempt at a full justification would necessitate a long digression. Essentially, in sub-phase 2*a* we have $\hat{Q}'$ constant (which means that dQ must be growing geometrically), whereas in 2*b* we only have dQ holding constant (so that $\hat{Q}'$ is falling).

otherwise have been expected. How long will depend on the size of $d\rho$. (The latter will of course stay positive until the switch to a Basic Expansion.) But if eventually the negativity of $d\alpha'$ wins out, $d\beta$ will become negative and then a crisis will have come. ***P'*** will fall, and speculators may panic and attempt to retrench. If they can ride this storm there are better times ahead as the Proportional Expansion is about to begin. [Whether they can or not will depend on how far out on a limb the speculators have already gone and whether they can manage to 'hang in there' without having to liquidate their stocks.] In anticipation of a later comparison by Lonergan let use call this first sub-cycle of minor boom to panic and possible crisis as *Kitchin 1*.

Proportionate Expansion. $[\hat{Q}'' = \hat{Q}'] : d\rho = 0$

Since $d\rho = 0$ we have $d\beta = d\alpha' + \rho\, d\alpha''$

Since it <u>is</u> an expansion neither $\dot{Q}$ can be zero. [for then they would <u>both</u> have to be zero, and we would instead be in a Static phase.]

This must mean that both $d\alpha'$ and $d\alpha''$ will be positive for a while, as the short-term acceleration develops.

During this period we will therefore have

$$d\beta > 0 . \qquad [\,(+) + (+)\text{x}(+) = (+)\,]$$

So β will be increasing.

However, $d\alpha'$ cannot continue to be positive [*i.e.*, greater than some non-zero number], for this would necessitate compounding of the effects on ***Q'***, giving rise as a geometric progression.[32] But this means that $d\alpha'$ will have to turn negative.

Because this turn round in $d\alpha'$ will take some time it is likely that α'' will be well along on its upward path to its maximum by the time it happens, so that $d\alpha''$ will be either zero itself or close to zero. Since we are still considering a Proportional Expansion, $d\rho$ remains at 0 and so we still have

[32] Lonergan has already discussed this in an earlier Section. See *CWL 15*, p. 120. See again footnote 28 above, and the related Appendix.

$d\beta = d\alpha' + \rho\, d\alpha''$. But this means that now $d\beta$ will either simply take its sign from $d\alpha'$ and be negative or at least follow it very soon after as any residual positivity in $d\alpha''$ is overcome.

Of course $d\beta$ will be even more negative if surplus production has also faltered, making $d\alpha''$ negative as well. In any case, $d\beta$ will become negative, so that the price spread will fall. As we have already argued, this will result in a fall in ***P'***. Speculators may again panic and attempt to retrench. This second instance of the boom, panic, crisis pattern will be *Kitchin 2*.

Basic Expansion. $[\hat{Q}'' < \hat{Q}']$ $d\rho < 0$

If the second crisis is survived, eventually $d\alpha'$ will return to a positive value, at the start of the Basic Expansion. Both α' and α'' will mount to their maxima, and once again there will be a minor boom.

But when they have reached this maximum they will stay there (*i.e.*, with $d\alpha' = 0 = d\alpha''$). This will mean that

$$d\beta = \alpha'' d\rho$$

But now that we have entered the Basic Expansion, this is negative (because $d\rho$ is negative). Speculators will wish to pull out as prices begin to fall. Both $d\alpha'$ and $d\alpha''$ will become negative. This third example of boom, panic, crisis will be *Kitchin 3*. In a reverse of the previous argument, a negative feedback may ensue, and the economy fall into a slump.

If at this stage the speculative feedback did not occur we would be entering the egalitarian phase of a pure cycle. But this is not what tends to occur in practice. Instead the signals are misread, and there is no recovery mechanism, and if no new genie can be pulled out of the hat we will fall into a full-blown depression.

As already anticipated, Lonergan has suggested the identification of his triple boom-and-crisis pattern with Schumpeter's three smaller cycles he called *Kitchins*, within one longer cycle (ideally a pure cycle but probably the 'trade' version) called a *Juglar*. He rejects (as non-economic, and probably entirely) the notion of an even longer '*Kondratieff*' cycle due to such things as technological change. If such an effect is in fact apparent in the data then it must be just a spurious pattern due to random effects. The only way one could argue to such a true long-period cycle would be on the basis of some 'theory of history', which would have its own presuppositions entirely outside the economic sphere. This is not the case with the *Kitchins* and the *Juglars*, which were derived by analysing the internal dynamics of the economy and its interactions with human adaptation (especially by way of adequate understanding).

Finally, recall that all of the above analysis assumed that there were no transfers from the Redistributive function (*i.e.*, ***D'- e'I' = 0***). [Otherwise we could not have argued from ***I'*** through to ***E'***.]

A speculative boom could occur because of a positive ***D' – e'I'*** . Alternatively it could happen by way of increases in the proportion of total income that goes to Basic Demand offset by increased ***D" – e"I"*** to counteract the effect of this on Surplus Demand. In either case the extra money amounts will permit the price spread to be maintained or reinforce its tendency to expand. In ***Rf***, however, this will appear in such results as a growing stock market. The misfortune of this is that if there is indeed an eventual *collapse*, it will be made worse, for the bigger they are the harder they fall.

Appendix: A Connection Between Growth Rate and Quantity.

(Recall again that we are using the more compact notation $\widehat{X}$ for the growth rate of a variable X, in place of Lonergan's $\frac{1}{X}\frac{dX}{dt}$).

Consider the case where $\widehat{X}$ is a positive constant.

<u>Verbal Statement:</u> If growth rate $\widehat{X}$ is a positive constant then X itself must be increasing in geometric proportion [*i.e.*, exponentially.]

<u>Symbolic statement:</u>

$$\dot{X} = k - 1 \Rightarrow X = k^{n-1}X_1 \text{ where } k > 1$$

<u>Proof</u>. Discrete case: (Take $dt = 1$)

$$\frac{dX}{X} = k - 1 \Rightarrow dX = (k-1)X$$

$$\Rightarrow X + dX = kX$$

$$\Rightarrow X_n = kX_{n-1}$$

$$= k(kX_{n-2}) = k^2 X_{n-2}$$

$$= \ldots\ldots = k^n X_1$$

Continuous case: It will be simpler to replace k - 1 by a, say.

$$\frac{1}{X}\frac{dX}{dt} = a$$

$$\frac{dX}{X} = adt$$

Integrate this over time: $\log X = at + C$

When $t = 0$ let $X = X_o$

So $C = \log X_o$ $\log X = at + \log X_o$

$$\boxed{\begin{aligned} &\log X = at + \log X_o \\ &\log X - \log X_o = at \\ &\log \frac{X}{X_o} = at \\ &\frac{X}{X_o} = e^{at} \end{aligned}}$$

Final result: $X = X_o e^{at}$

<u>Common example.</u> <u>*Compound Interest (C.I.)*</u>.

Let the interest rate, expressed in decimal form, be r .

The rule for *C.I.* can be written as $A_{n+1} = (1+ r)A_n$ where A_n is the 'Amount' held in year n.

Applying this recursively gives the usual formula: -

$A_{n+1} = (1+ r)^n A_1$ [The traditional form is $A = P(1 + r)^n$]

This formula applies if the growth rate (the interest rate) is r .

[Simply replacing $1 + r$ by k gives the same result as in the discrete case above.]

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