Comparison of Autoregressive Moving Average and State Space Methods for Monthly Time Series Modelling of Labrador and South-East Quebec Rivers

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Abstract

Time series data such as monthly stream flows can be modelled using time series methods and then used to forecast flows for short term planning. Two methods of time series modelling were reviewed and compared; the well-known autoregressive moving average (ARMA) method and the State- Space Time-Series (SSTS) method. ARMA has been used in hydrology to model and simulate flows with good results and is widely accepted for this purpose. SSTS modelling is a method that was developed in the 1990s and is relatively unused for modelling river flow time series data. The work described in this paper focuses on modelling the stream flows from basins of different sizes using these two time series modelling methods and comparing the results. Three rivers in Labrador and South-East Quebec were modelled; the Romaine, Ugjoktok and Alexis Rivers. These rivers are located in various areas of the study region, having different drainage aspects and differing basin sizes. Both models were compared for accuracy of prediction, ease of software use and simplicity of model to determine the preferred time series methodology approach for modelling these rivers.

Keywords: ARMA models, Labrador, Monthly river flows, Quebec, State-Space Models..

Introduction

In the field of water resources, stream flow analysis is used to determine if flows are sufficient and reliable for a project. For developing a run-of-river hydroelectric project, for example, the engineer uses stream flow analysis to determine whether a stream can meet the energy demand throughout the year. In addition, short term forecasting can be used to help manage water at future dam facilities. Part of this design process includes developing a stream flow model based on historical flow records, simulating flows from the model to determine whether the model provides a good representation of the historical flows and then in some cases, using the model for short-term forecasting.

This paper compares two methodologies for modelling stream flows in Labrador and South-East Quebec: Auto Regressive Moving Average (ARMA) and State-Space Time-Series (SSTS). The ARMA or Box-Jenkins methodology (Box et al, 2008) is well known and has been used in modelling of hydrometric time series since the early 1970s. As for the SSTS methodology (Harvey, 1989; Commandeur and Koopman, 2007), it is relatively new, and has rarely been used for hydrological modelling. It has been used primarily for economic time series model development and forecasting. The SSTS methodology is also referred to as a Structural Time

Series approach. One of the principal purposes of the work described here is to assess SSTS analysis as an additional tool for hydrologic time series modelling.

Preliminary analysis of the data

Figure 1 depicts the locations of 79 hydrometric stations in Labrador and South-East Quebec; along the portion of Quebec between Labrador and the Gulf of St. Lawrence. The length of record at these stations ranges between 8 and 60 years, some stations being on regulated rivers and some not. For the purposes of this study, the criteria for selecting rivers of interest were that they should be non-regulated, have more than 15 years of data, and have as little missing data as possible. In addition to these initial screening criteria, rivers with different basin sizes (as identified upstream of each gauge), basin aspects and geographic locations were preferred for the comparison of the two methodologies. All data were taken from Environment Canada (2013) and the Government of Quebec (2013). The study area was divided into two subareas; North and South of Goose Bay. These geographic areas were chosen to represent different soil and vegetative conditions. As a result, an effort was made to select rivers from both north and south regions. In addition to geographic location, it was desirable to select rivers of various drainage basin size as well as drainage aspect.

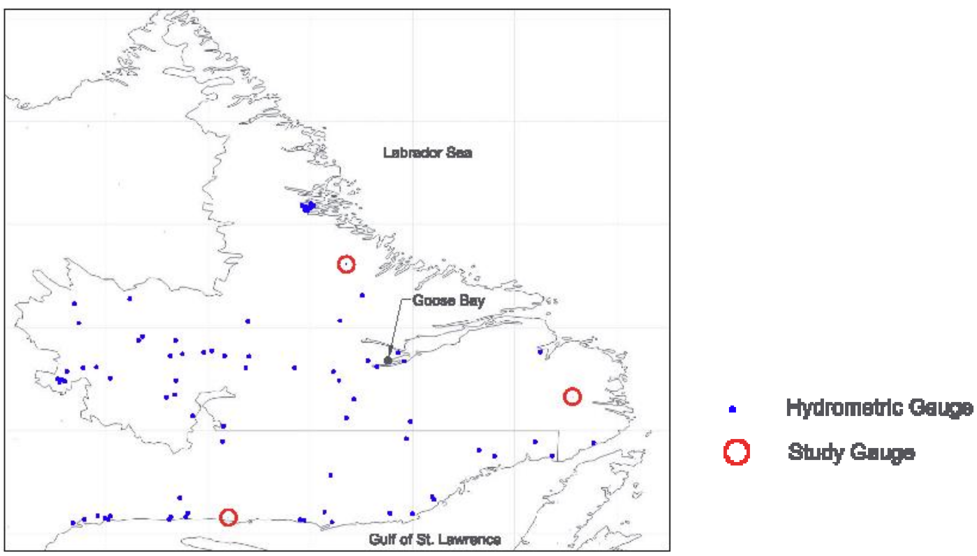

Figure 1: Hydrometric stations in Study Area (Environment Canada, 2010)

Although the availability of records from 79 gauge locations seems promising, most did not meet the screening criteria. Only 12 remained from the list. These were reviewed based on geographic location, drainage aspect and drainage size, and the final selection included one from the northern region, and two from the south, one draining to the Gulf of St. Lawrence, and two to the Labrador Sea. In the northern region Ugjoktok was selected over Kanairiktok since it has the longer record of the two rivers. This river drains to the Labrador Sea and has a basin size of 7570km^2. In the southern region, with rivers draining towards the Gulf of St. Lawrence, the records from six rivers met the screening criteria. Since the Romaine station has 46 years of data it was selected for further study. This river has a basin size of 13,000 km^2. Of the remaining rivers in the southern region draining towards the Labrador Sea, the Alexis station was selected in preference to Eagle River to provide a record from a basin with a small drainage area compared with Ugjoktok and Romaine. The records from Alexis, Romaine and Ugjoktok Rivers were ultimately selected for study. Table 1 summarizes the information for these selected rivers and their locations are shown as red circles in Figure 1.

Table 1: Selected rivers for time series modelling

River	Drainage Area km^2	Years of Data	Location	Drainage Basin Aspect	Outlet location
Ugjoktok	7570 (medium)	32	North	West/East	Labrador Sea
Alexis	2310 (small)	34	South	West/East	Labrador Sea
Romaine	13000 (large)	56	South	North/South	Gulf of St. Lawrence

The records selected for this study therefore include one small, one medium and one large river. Each river has more than 32 years of predominantly continuous data. One stream is located north of Goose Bay and two are located south with the two south streams having differing basin aspects and outlet locations. This diversity of location, basin orientation and outlet location was selected to cover the principal features that might affect stream flows and thus the models.

Methodology

Two methodologies for modelling and predicting stream flows were used: ARMA and SSTS. The software Structural Time Series Modelling Program (STAMP Version 8.3) developed by Koopman et al (2009) and sold by Timberlake Consultants Ltd. was used for the state-space time-series modelling. The ARMA modeling was completed using the Minitab 16 software (Minitab Inc, 2013). The purpose was to compare the two modelling methodologies to determine accuracy of the prediction as well as ease of use of the software for the practicing hydrotechnical engineer.

Prior to modelling, the monthly flow data for each of the three rivers were collected and reviewed for normality using normal probability plots. A review of all three time series indicates that the data sets are not normally distributed. The log-transformed data sets, however, were normally distributed and the normal plots showed no outliers. As such, the transformed data for each river were used for modelling. Time series plots were completed for all rivers. As shown in Figure 2, there appears to be no trend in the Alexis River time series. To verify, the data set was statistically tested for trend using simple regression of the logged flows with time. For the Alexis River, the trend is not statistically significant at the 5% level. Testing showed that none of the three rivers had significant trend.

As with many streams and rivers in this area, seasonality is expected. To verify seasonality, autocorrelation (ACF) plots for each river were prepared. As shown in Figure 3, for the Alexis River, autocorrelation is significant and seasonality is confirmed based on the sine wave appearance of the plot which repeats every 12 months. Once each data set had been reviewed as described above, the monthly flows for the three rivers were modelled using the two methodologies. For the purposes of illustration, only the results for the Alexis River are included in this paper. The results for the other rivers were similar and will be shown during the conference.

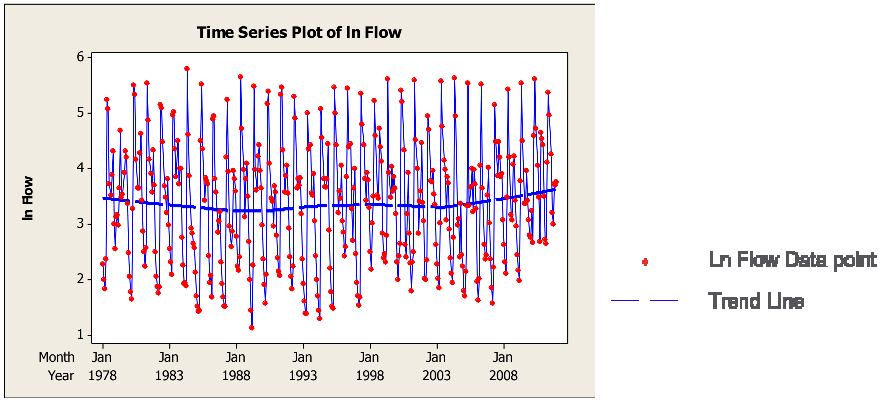

Figure 2: Time series plot of the Alexis River

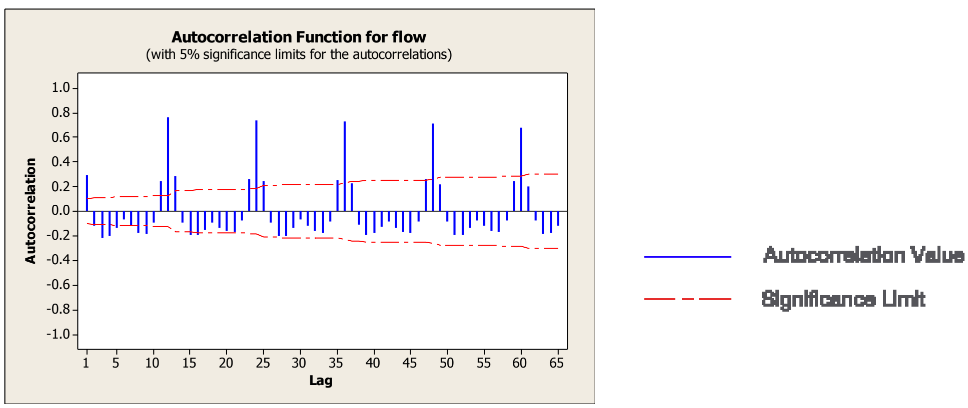

Figure 3: Autocorrelation function of Alexis River monthly flows

ARMA with Harmonic Analysis

When modelling monthly stream flows with seasonality, ARMA models provide better results when the data set is deseasonalized. For this reason, the seasonal component, for each of the three rivers, was modelled using harmonic analysis and the residuals were then modelled using the ARMA methodology. The two models can then be used together to generate new data sets for comparison to the actual data set. The summary statistics for the actual and generated data sets can be compared to confirm the model's ability to simulate flows that are within specified confidence intervals. The model can then be used to forecast stream flows up to several months in advance. In the case of the Alexis River, the seasonality was modelled using harmonic analysis by fitting a regression equation using sine and cosine pairs (Box et al, 2008). Since the largest period is 12 months, a maximum of 6 sine and cosine pairs can be used in the equation and thus models using 1 through 6 pairs were checked to determine which regression equation best fit the seasonal component. The best fitting model is selected based on statistical significance of the regression, as well as a low residual error and a high adjusted R^2, and as such it was determined that the equation with 6 sine and cosine pairs was the best fit for the Alexis data. As with regression, the assumptions of ANOVA were checked and found to be acceptable; primarily that the residuals are normally distributed, have constant variance and are independent. The general harmonic analysis equation is as shown in Equation (1).

$$Z_t = \alpha + \beta_0\sin(2\pi ft) + \beta_1\cos(2\pi ft) + \beta_2\sin(4\pi ft) + \beta_3\cos(4\pi ft) + \beta_4\sin(6\pi ft) + \beta_5\cos(6\pi ft) + \beta_6\sin(8\pi ft) + \beta_7\cos(8\pi ft) + \beta_8\sin(10\pi ft) + \beta_9\cos(10\pi ft) + \beta_{10}\sin(12\pi ft) + \beta_{11}\cos(12\pi ft) + y_t \quad (1)$$

where Z_t is the time series, α is the regression constant, β is the coefficient for each sine or cosine term and $2\pi f$ is a constant with $f = 1/12$ for monthly values, and y_t are the residuals.

This model can also be used to reasonably forecast up to 6 months in advance although the primary purpose of the model was for flow simulation. This model was used to predict the last 5 years of data for comparison with the actual values. As shown in Figure 4, the model without the random term does a reasonable job of predicting flows for Alexis River.

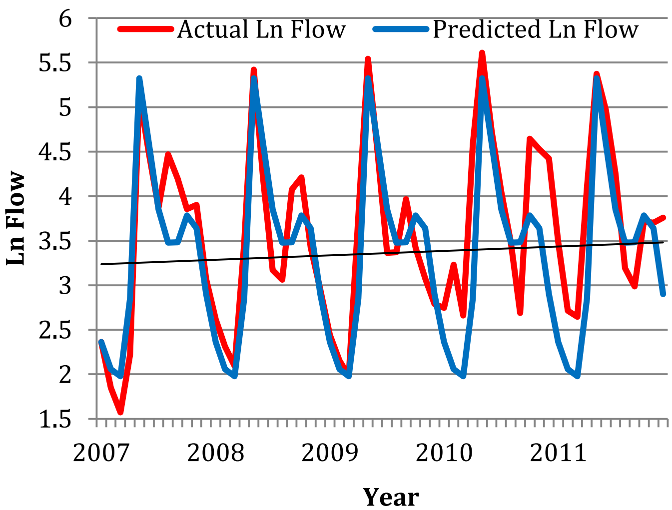

Figure 4: Actual and Predicted Flows from Harmonic Analysis for Alexis River

Once a seasonal model was selected, the autocorrelation and partial autocorrelation of the residuals were plotted (see Figure 5). These plots provide guidance on which ARMA model may be the best fit of the residuals. For example, if there is only 1 significant lag on the ACF plot then the best model is probably AR1. If there are two significant lags on the ACF plot, then AR2 is probably the best model. For the Alexis river, there is a sigificant lag 1 on both plots as shown below. This would suggest that an AR1 model might be the best fitting model. As indicated, interpretation of the ACF and PACF plots is an important tool to help guide the determination of the best ARMA model. The residuals from the harmonic analysis were modelled for Alexis using an AR1 model. ACF and PACF plots were also completed for the AR1 model residuals and they indicated that the model has addressed the significant correlations; thus the AR1 model is suitable for modelling the residuals of the deseasonalized data. The general AR1 model equation is given by Equation (2).

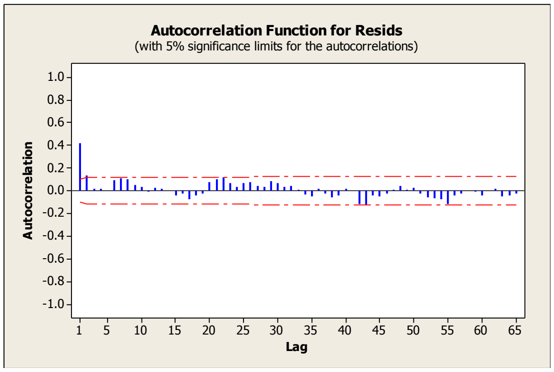

Figure 5a: ACF of residuals

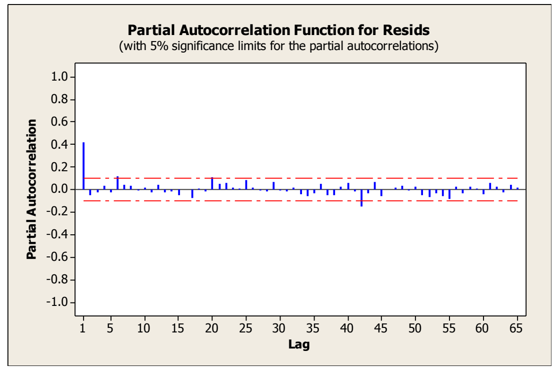

Figure 5b: PACF of residuals

$$y_t = \mu + \rho(y_{t-1} - \mu) + \varepsilon_t \qquad (2)$$

where y_t is the residual from (1), a time dependent series, and ε_t is the independent series, μ is the mean of y_t, and ρ is the AR1 coefficient. To simulate flows, the model must be verified to demonstrate that it can indeed generate data sets of the same sample size that on average have similar historical statistics. The mean, standard deviation and lag 1 correlation (r1) for the generated data should be within a few percent of the actual statistics. For the Alexis River, these are within 2% of the historical values. Based on the model's ability to adequately reproduce the mean, standard deviation and the r1, in conjunction with the ACF and PACF plots of the AR1 model residuals, it is determined that the AR1 or ARMA (1,0) model in conjunction with the harmonic seasonal model is an overall reasonable model for the Alexis River.

State-Space Time-Series Analysis

The STAMP software applies the state-space time-series modelling theory described in Harvey (1989) to explain each of the time series components. The components include level (roughly equivalent to intercept in a regression equation), slope, seasonality, cycle and irregular or white noise. Each component can be specified as fixed or stochastic as required; fixed meaning that the component baseline does not change over the time period and stochastic meaning that the baseline varies over the time period. For example, the state equation for a model with level, seasonal and irregular components is given by Equations (3) to (15) below.

$$Z_t = \mu_t + \gamma_t + \varepsilon_t \qquad (3)$$
$$\mu_{t+1} = \mu_t + \xi_t \qquad (4)$$
$$\gamma_{1,\,t+1} = -\gamma_{1,t} - \gamma_{2,t} - \gamma_{3,t} - \gamma_{4,t} - \gamma_{5,t} - \gamma_{6,t} - \gamma_{7,t} - \gamma_{8,t} - \gamma_{9,t} - \gamma_{10,t} - \gamma_{11,t} - \omega_t \qquad (5)$$

$$\gamma_{2,\,t+1} = \gamma_{1,t} \quad (6) \qquad \gamma_{6,\,t+1} = \gamma_{5,t} \quad (10) \qquad \gamma_{10,\,t+1} = \gamma_{9,t} \quad (14)$$
$$\gamma_{3,\,t+1} = \gamma_{2,t} \quad (7) \qquad \gamma_{7,\,t+1} = \gamma_{6,t} \quad (11) \qquad \gamma_{11,\,t+1} = \gamma_{10,t} \quad (15)$$
$$\gamma_{4,\,t+1} = \gamma_{3,t} \quad (8) \qquad \gamma_{8,\,t+1} = \gamma_{7,t} \quad (12)$$
$$\gamma_{5,\,t+1} = \gamma_{4,t} \quad (9) \qquad \gamma_{9,\,t+1} = \gamma_{8,t} \quad (13)$$

where μ_t is the level component and γ_t is the seasonal component with (s-1) state equations where s is the periodicity of the seasonal component.

The calculations are complicated so the evaluation of SSTS as a practical method necessarily includes consideration of the software available for the analysis. The software program STAMP makes it easy for a user to enter the data and quickly develop a model. To select the best fitting model, the model results must be reviewed and the diagnostic test values must fit within critical values. The software does not provide the critical values for the diagnostic tests so a spreadsheet was developed to calculate these values and to compare a number of models to find the best fit. For the three rivers, not all the diagnostic tests were within the critical values and as such the model meeting the most diagnostic criteria was selected. To diagnose the suitability of each model, the software provides a results page displaying test statistic results. Figure 6 is a sample of the results for one of the Alexis River models. Plots are easy to generate, and as Figure 7 for Alexis River shows, the time series can be broken down into separate plots illustrating level, seasonality and irregular components. The irregular component plot shows no pattern and thus confirms that the seasonal component has been captured in the seasonal equation. Also, the level component is stochastic as illustrated in the plot since the level value changes over time.

Estimating...
Very strong convergence relative to 1e-007
- likelihood cvg 1.05026e-015
- gradient cvg 5.03764e-010
- parameter cvg 1.65994e-008
- number of bad iterations 0
Estimation process completed.

UC(10) Estimation done by Maximum Likelihood (exact score)

The database used is E:\alexis.in7
The selection sample is: 1978(1) - 2011(12) (T = 408, N = 1)
The dependent variable Y is: Lflow
The model is: Y = Level + Seasonal + Irregular
Steady state. Found

Log-Likelihood is 215.646 (-2 LogL = -431.293).
Prediction error variance is 0.294902

Summary statistics	Lflow
T	408.00
p	2.0000
std.error	0.54305
Normality	19.158
H(132)	0.71624
DW	1.3482
r(1)	0.32322
q	24.000
r(q)	-0.019666
Q(q,q-p)	87.041
Rs^2	0.15993

Variances of disturbances:

	Value	(q-ratio)
Level	0.00416959	(0.01742)
Seasonal	4.25131e-005	(0.0001776)
Irregular	0.239400	(1.000)

Figure 6: Sample output from STAMP software

Completing multiple models is easy when a software package is used, but the difficulty lies in diagnosing each model. Unlike the diagnostic tests for the illustrated ARMA modeling which are well prescribed, the SSTS software diagnostic test results do not provide the acceptable critical value or the p-value for each of the test statistics. As a result, determining the suitability of a model was difficult and time consuming. It was also challenging to determine from the results how to modify the model to achieve the best fit. Reviewing ACF and PACF plots help direct the user to the appropriate model by quickly showing in graphical form if there are significant correlations remaining in the model residuals. Unlike the ARMA modelling process where ACF and PACF plots are easily generated from the model residuals, the SSTS software requires the user to store the residual values from the model and then plot each of the ACF and PACF plots. Review of the plots that are automatically generated for each model does not provide enough help on determining how to adjust the model, and as such the modeller is left to try various combinations of components and then review the diagnostic test results for each. As shown in Table 2, the X marked fields indicate the model run combinations that were completed to determine the best model for Alexis River. There were 9 models developed, each with diagnostic results that were separately reviewed in order to select the best model. The critical values were calculated in a separate spreadsheet and Table 3 gives a summary of the diagnostics and the critical value calculations that were completed to help select the best model. Table 3 shows that model 9, which is comprised of a stochastic level term, a stochastic seasonal term, a short term cycle term and an irregular term as seen in Table 3, meets most of the criteria and has a very strong convergence. The assumption of normality is not met, but this is the best model of all model runs and as such, was ultimately selected for Alexis River. The software does not allow for simulation of the model to verify summary statistics, however, the model could be used to predict a period within the actual data time series and compare to the actual data. As shown in Figure 8, the model does a reasonable job of predicting values for the last 5 years.

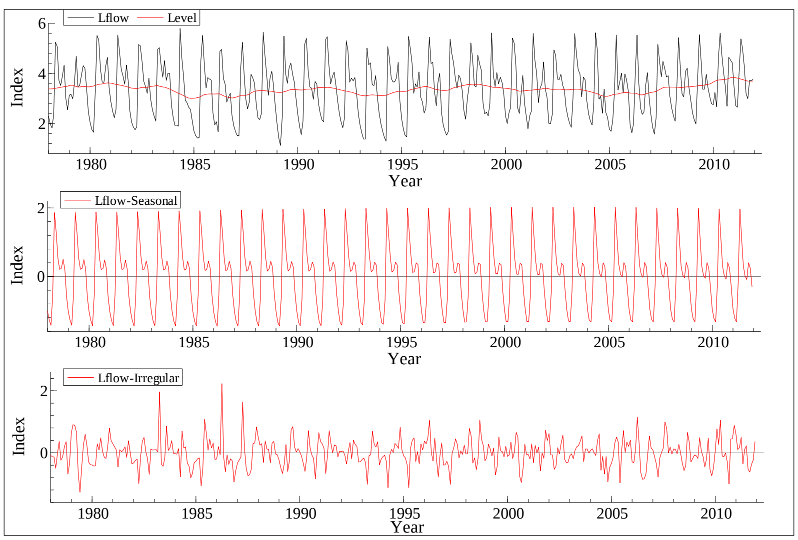

Figure 7: Plots of level, seasonal, and irregular components for Alexis River

Discussion of Results

Both methods appear to produce models that reasonably predict the actual monthly time series data for the Alexis River as can be seen in Figures 4 and 8. In fact, for all three rivers selected in this study; Alexis, Romaine and Ugjoktok Rivers, the location, basin size and flow directions did not seem to affect the choice of models. Model structures for all 3 rivers were similar whether the ARMA or SSTS methods were used. There are, however, pros and cons for each of these methodologies. These are listed in Table 4. Since these methods require computer programs to carry out the calculations, the pros and cons of the methods for the practicing engineer include consideration of the available software.

Table 2: Combinations of structural models attempted for the Alexis River

model run	level		slope		seasonal		cycle		irregular
	deterministic	stochastic	deterministic	stochastic	deterministic	stochastic	short	medium	
1	X								X
2		X							X
3	X		X						X
4		X		X					X
5	X				X				X
6		X				X			X
7		X	X			X		X	X
8		X	X			X	X		X
9		X				X	X		X

Table 3: Summary of diagnostics and critical values for models compared for the Alexis River

Model	r(1) and r(q)		Q(q,q-p)		H		N		AIC	Convergence	steady state
	crit value	assumption	chi square crit value	assumption	f crit value	assumption	chi square crit value	assumption			
1	0.099015	not met	35.172	not met	1.3286	met	5.991	not met	84.02		found
2	0.099015	r1 met, r24 not	35.172	not met	1.3286	met	5.991	not met	39.73	very strong	
3	0.099015	not met	33.924	not met	1.3286	met	5.991	not met	99.71		found w/o full
4	0.099015	r1 met, r24 not	33.924	not met	1.3286	met	5.991	not met	46.64	very strong	found w/o full
5	0.099015	r1 not met, r24 met	33.924	not met	1.33	met	5.991	not met	-409.26	patt in irregular	
6	0.099015	r1 not met, r24 met	33.924	not met	1.33	met	5.991	not met	-431.22	very strong	found
7	0.099015	both met	28.869	met	1.33	met	5.991	not met	-477.98	weak	found w/o full
8	0.099015	both met	28.869	met	1.33	met	5.991	not met	-477.98	strong	found w/o full
9	0.099015	both met	30.144	met	1.33	met	5.991	not met	-491.81	very strong	found

Table 4: Pros and Cons of ARMA using Minitab and SSTS Methodology using STAMP

ARMA with Harmonic Analysis using Minitab	State-Space Time-Series model using STAMP
PROS	
Results indicate whether diagnostic tests are significant so user can quickly diagnose the model.	Seasonality can be included in a single model.
ACF and PACF plots for the model residuals are produced with the model results so user can easily decide how to improve the model.	Modelling is very fast and easy to do.
Simulation of the data can be completed to verify the model accuracy	
CONS	
Monthly data must be deseasonalized prior to ARMA modelling.	Diagnosis of the model is very time consuming and requires knowledge of diagnostic tests, some of which are specific to this program.
Simulation is time consuming and requires macros to perform the simulation, but can be done.	Multiple runs are required to verify diagnostics before the best model can be selected
Harmonic analysis is not built into the software and a number of macros are required to complete the seasonal modelling component.	No easy way to simulate the model to verify that the summary statistics for the model are close to the actual summary statistics. Simulation can be programmed using a separate software module.

Conclusion

Both methods are effective ways to develop models of stream flows, however for the practicing engineer, the tools make a difference. Although it is more time consuming to develop an ARMA model in Minitab than to develop a SSTS model in STAMP, it is relatively easy to simulate the flows using the ARMA model. In addition, the ARMA or Box-Jenkins methodology is well known, widely used in hydrology, and the statistical principles are easier to follow. Furthermore, there are many other software packages available for ARMA modelling and the software packages available for SSTS modelling are limited. The main difficulty with developing a SSTS model is in the diagnostics and the theory behind the state-space approach is not easy to follow for a typical hydrotechnical engineer. If features were added to STAMP, or alternative software for SSTS analysis, such as automatic graphing of residual plots, providing critical values for diagnostic tests, providing guidance for the interpretation of diagnostics in the model results, and adding provision for simulation, the program could be a very good tool for the practicing hydrotechnical engineer wanting to develop a SSTS model to simulate and forecast stream flows.

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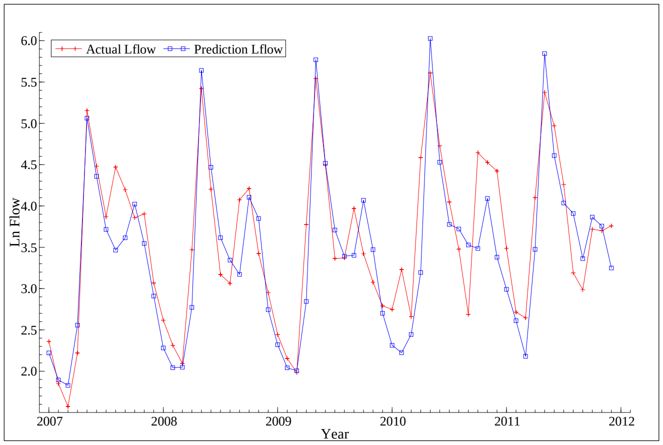

Figure 8: Actual and predicted flows from the SSTS Model 9 for Alexis River.