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THE TWIN PARADOX: WORKING TOWARD FUNCTIONAL INTERPRETATION

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Introduction

In the early 20th century, Albert Einstein discovered the Special Theory of Relativity (c.1905). After about a decade of work,[1] he followed with the General Theory of Relativity (1915). In addition to offering new account of certain otherwise unexplained phenomena, his results called for a revision of previously held notions of space and time. Indeed, this led to assertions not only of what was puzzling, but what in some cases seemed paradoxical. Now, it is not the purpose of this short article to enter into a lengthy analysis of Einstein's Theories of Relativity. Instead, I focus on the traditional and allegedly rigorous argument that is prior to, but leads to, the famous Twin Paradox. That prior argument gives the following scenario: A twin leaves the earth at a speed that is some considerable fraction of the speed of light. The argument then uses equations from the Special Theory of Relativity to deduce that time for the traveler is slowed down; consequently the traveler who returns to the earth would be younger than the twin who stayed at home.[2]

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[1] For the geometry of Riemann, Ricci and Levi-Civita, Einstein consulted with the mathmetician Grossman. Also, after close communication with Hilbert, both Hilbert and Einstein came to versions of a General Theory within just a few days of each other.

[2] For one of Einstein's discussions of the matter, see his *Vierteljahrachrift der Naturforsh, Gesellesch. in Zurich,* 56 (1911) ; see also August Kopff, *The Mathematical Theory of Relativity* (trans. H. Levy.

The Twin Paradox is not a new topic. What is new in this article is that it is an exercise toward interpretation that is *functional*, in the sense discovered by Lonergan in *Method in Theology*. The author being interpreted is P. Tipler; and the primary document[3] is taken from his well known textbook.[4] I try to lay out the basic argument in a way that reveals the operative insights, as well as the significant oversights. As it turns out, it would seem that there is neither theoretical basis nor experimental evidence for the alleged “slowing of time” for the high speed traveler.[5]

Within the main outline requested by the Editor,[6] some details on the division of the article are as follows: Section I consists of a few remarks toward expressing my Personal Context, and is intended to be a preliminary attempt toward being in keeping with the dialectic process described on p. 250 of *Method in Theology* This is complemented by the last Section IV, where I look specifically to physics and add what, for this paper, might be called a Context of Concern. For, as is revealed in the course of the paper, there are numerous fundamental issues that arise, calling for further and prolonged attention. Section IV is not a controlled effort within the hodic process. All the same, given the fact that it is early days yet for functional specialization (there is not yet a functioning

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London: Methuen, 1923), p. 52

[3] I am thinking of the distinctions between documents that are primary, secondary and tertiary, as described by Lonergan in ‘The Sketch,’ Sec. XVII.3.6 of *CWL 3*.

[4] Paul Allen Tipler, *Physics for Scientists and Engineers, 4th ed.*, Volume 3 *Modern Physics: Quantum Mechanics, Relativity, and The Structure of Matter* (New York W.H. Freeman/Worth Publishers, 1999), 1258-9. This contemporary text gives the main features of the standard argument; and over the last almost thirty years, it has been widely used by numerous North American universities for undergraduate physics classes. I hoped, therefore, that it would be an easily accessible document for readers of the present article. Henceforth, this text will be referred to as Tipler.

[5] Strictly speaking, this last comment is evaluative in a way that may make it belong not to Interpretation, but to Dialectic. I leave it in the paper, however, for reasons that I give in the last paragraphs of this introduction.

[6] The Editor’s request was that contributions to the present volume be divided according to Personal Context, Content and Context. See the editor’s Introduction in this volume.

community as such), perhaps the various remarks will be useful for later re-cycling.[7] In Section II A, I briefly indicate the physics context for the time-paradox; and then move immediately to Section II B, which provides the primary document. In Section III A, I give hypothetical expression that is intended to give the main content of the argument for the paradox. In Section III B, I try to identify certain key oversights of what was expressed in IIIA.

Working on this paper led me to methodological questions. For (not to exclude conversations between specialties), it seems to me that a prime directive for the Interpreter is to communicate the meanings of authors to Historians. What, though, if an author being interpreted holds a mistaken view, one say that is not compatible with the norms and exigencies of their field? Again, what if the view of the author being interpreted is grounded in a counter-position?

Speaking strictly as an interpreter, then, would one not comment directly on the validity of the arguments leading to The Twin Paradox? Instead, the interpreter might relay the essential meaning of the author to historians, in the area of expertise. (Both interpreter and historian would, therefore, need to be up on details of the field.) The interpretation could, for example, involve an axiomatic presentation of the author's hypotheses (explicit and implicit), and a plausibility argument for the author's view (whether or not the view is ultimately correct). Within a context of universal viewpoints, the interpreter might then communicate a best effort toward a non-critical and pure formulation of the author's work. Identifying and resolving problematic results might then involve dialectic work as a basis.

As it happens, I have not followed that pattern in this paper; and I also leave an evaluative component in the article. My reasons are as follows: 1. The argument for The Twin Paradox has been in general acceptance long enough; and 2. To leave out in the open the fact already mentioned, that this is a first effort toward functional interpretation in physics, and that the problem of how to interpret a mistaken view will need to be

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[7] See also the last paragraph of this Introduction.

worked out by future functional (collaborative) efforts.

I Personal Context

In university, I would have gone into theoretical physics, but the University of Toronto physics program required upwards of 20 hours a week of lab based time, this in addition to the full retinue of courses in mathematics and theoretical physics. In order to avoid that laboratory work detail, I entered the mathematics program, with the plan to take selected physics courses as possible.

I became immersed in the world of mathematics; the slow learning; the doing; later also the teaching; and more recently I have been making an effort to appreciate details of the method of mathematics[8] (*method* in the sense discovered by Lonergan).

Some documents that took my early attention include *The Odyssey* (Fitzgerald tr.); class notes from an inspired high school physics teacher; an unusual Calculus text that I found in a second hand bookstore; Chesterton's *The Dumb Ox*; and piano sheet music of Beethoven, Brahms and Chopin. In the early years of university study, I was aware also of Lonergan's work. On good authority I was given to trust that his work was radically important, but I had not yet managed any prolonged reading.

My first experiences with music were happy beginnings. Later "music lessons" were not all so positive. Nevertheless, there was some continuity preserved, leading to the later pleasure and joyful humility of trying to play certain piano pieces of the three greats just mentioned - B, B and C. In some way that my young self could make some sense of, Fitzgerald's translation of Homer's *Odyssey* helped me grow somewhat in appreciation of the wonder-drama that is daily journeying. Chesterton helped me get a real sense that Aquinas was a teacher whose work should be taken seriously, whatever field I was headed for. At the same time as chancing upon these various documents, I also was becoming increasingly aware of the Teacher Tri-Friends.

From the beginning of my university studies, Lonergan's

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[8] See my "Reflections on Progress in Mathematics," *JMDA* 3 (2003), 97-116.

work has gradually become central to my efforts in scholarship. Not all good teachers need to be deceased! I have been lucky in scholarly help received from two brothers, James and John (dec.), both of whom had studied Lonergan and were talents in (among other things) academic honesty. John introduced me to Phil McShane, whose teaching and writings have been of continued and immeasurable help.[9]

I have tried to make some sense of contemporary mathematics and physics. Lonergan's book *Insight* has been as massively opaque as it has been helpful for beginning to realize the necessity, in any discipline, of what Lonergan called "generalized empirical method" (*3 Coll* 141). Elementary exercises in mathematics and physics have helped me reach initial existential displacements with regard to experience, knowing and doing and the elements thereof.[10] Based especially on the effort to come to terms with certain problems in contemporary physics, I have found it possible to enrich, extend and refine initial displacements.

To place myself in mathematics, my background includes the standard repertoire of graduate courses (analysis, algebra, differential geometry, algebraic topology, homological algebra, to name a few). I have managed some rather modest contributions in C*-algebras, symmetries of differential equations, and applications of stability theory to biology population models. I have become familiar with the non-commutative differential geometry of Alain Connes.[11]

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[9] Sadly, I have found no academic help from "Lonergan conferences" that I have attended. I have been frustrated by the many talks and professional discussions that did not seek (or promote) theoretic understanding, in any stage of meaning. Instead, the emphasis was of a type of confining and merely linguistic work that runs counter to the expansive and enriching scientific objective. It seems to me then, that largely what has been going on in the name of "Lonergan studies" would, in mathematics say, compare to a group of scholars learning and comparing certain mere symbolic techniques, without fostering or reaching mathematical understanding.

[10] See the zero and fourth words of metaphysics: W0, W4, Philip McShane, *Cantower* XXIV, www.philipmcshane.ca.

[11] The "non-commutativity" is because, from the beginning, the theory ties together operator structures and geo-topological manifold structures; and operators in general do not commute: see Alain Connes,

In the winter of 1992-1993 I had the good luck of being able to attend a six month seminar at the Dublin Institute for Advanced Studies. Using his first book[12] as a basis for the lectures, Lochlainn O'Raifeartaigh gave an introduction to field theory in physics. At the time I was missing a lot of background, and so much went past me, or had to be noted for future reference. But the context took hold of me, as did the equations and diagrams for the "Lie algebras of particles". I found Lochlainn's civilized and intelligent presence inspiring. It was manifest that this kind man knew what he was talking about, and that he was serious in his commitment to understanding. The directions and mood of that seminar stayed with me, and have been part of what I have been trying to climb to since that time. While life and professional circumstances[13] have been "slowing me down", in 1996 -2000 I was finally able to begin honing up on some of the mathematics that I was wanting for my follow up into contemporary physics, namely, Lie Groups and Differential Equations.[14]

Over the last years I have dabbled in quantum mechanics,[15] and have been trying to keep abreast of main

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Noncommutative Geometry (San Diego: Academic P, 1994). In view of recent GUTS in physics, it is possible that results of this general type will be relevant to modern physics and real geometry. See Section IV, below.

[12] Lochlainn O'Raifeartaigh, *Group Structures of Gauge Theory* (Cambridge and New York: Cambridge UP, 1986).

[13] Teaching responsibilities have been extensive, but also have been a basis for happy growth. In the main, however, I have found the university environment to be hostile to human growth, for both students and faculty alike – which is especially sad, since the professed mission is education.

[14] P.J. Olver, *Applications of Lie Groups to Differential Equations*, (New York: Springer-Verlag, 1986/1993). This is an expository text. There are orientation problems, however, revealed in various explicit dismissals of the importance of what in fact are higher mathematical viewpoints. In particular, there is a lacking of adequate attention given to the geometric dimensions of the work together with an over-emphasis on mere algebraic technique. I did, however, find the text a convenient source of examples, and a useful introduction to the 20th century results on symmetries, generalized symmetries, the work of Emmy Noether, etc. The author also gives an extensive bibliography.

[15] T. Quinn, "From Schrodinger to Dirac: On Relations and Statistics", B.N. Prasad Centenary Commemoration, *Bull. Allahabad Math. Soc.*

directions in quantum field theory and quantum chromodynamics. My hope is to soon begin a more detailed work on the foundations of real geometry (see Section IV, below), with results increasingly oriented within the dynamic of hodic control. Of course, by definition, functional specialization is a community project, so part of my hope also is for the emergence of collaborative projects.

The first and second words of metaphysics regard what McShane has called the "aggreformic". For me, this remains mainly heuristic. At this time I do not know enough of the higher sciences to allow for significant detailed reflection on higher forms. The third word of metaphysics regards functional specialization.[16] As referred to above (see footnote 8), I have made a modest beginning toward recognizing the need for functional specialization in mathematics. In the main, however (and as is a familiar experience in science), the field remains richly and invitingly obscure.

Certainly, there is the potential, personally, for a more adequate *Assembly* and so on, that would no doubt reveal oppositions, affinities, sources. And related to the question of possible opposing horizons, there has been my slow and oftentimes problematic growth within certain differentiations. At the same time, I am not aware of having felt that there were any "necessary separations" between these different worlds. So, I have taken the Divinity, theology, metaphysics, science, the daily drama of journey, music, and so on, to be all of a piece. In that sense, I may refer to a spectrum of blending affinities that contributes to, and is somewhat unified within, my on-going efforts (such as they are) toward hodic oriented theoretic understanding.

Finally, my experiences in mathematics, physics and music have helped make undeniable the need for taking one's time and starting with elementary instances. As Klein wrote, "slowly to higher things."[17] In particular, I have found that

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(Indian Journal of Mathematics), Vol. 15, 2000, 69-100. This article was my first attempt at identifying some of the key insights (and oversights) in the works of Schrodinger, Heisenberg and Dirac.

[16] W3, *Cantower* XXIV.

[17] F. Klein, *Elementary mathematics from an advanced standpoint:*

pondering over the contemporary efforts of science to understand the familiar human experience of "space" and "time" can be an immensely enriching and likewise be a happily humbling exercise. Moreover, it is increasingly evident to me that such work can serve as a vital and crucial "bridge" (*CWL 3* 1992) to further issues.[18] This brings me, then, to the present article.

IIA The Context

What follows are Einstein's postulates and the Lorentz equations. The traditional argument that gives the Twin Paradox takes these as given.

Einstein's two postulates for his Special Theory[19] are:

1. Physical laws and principles are of the same form in all inertial systems, that is, in all reference systems which differ only in the fact that they are moving with constant velocity with respect to each other.
2. The velocity of light has the same value in all inertial systems.[20]

For the traditional calculation of the Lorentz Transformation Equations between two inertial frames F and F′, one assumes a (local) affine linear transformation, and that at $t = 0 = t'$, the two origins coincide. Invoking the second postulate, it is possible to then determine the coefficients of the transformation. For one space dimension x and time t, these equations turn out to be:

Arithmetic, algebra, analysis, trans. E.R. Hedrick and C.A. Noble, from the third German edition (New York: Dover Publications, 1925), 268.

[18] Clues to a fuller context are expressed in McShane, *Cantower* XII, "A Problem of Interpretation Arises," and *Cantower* XXXI, "Time and Distance: Feynman I, ch. 5; *Insight*, ch. 5."

[19] English translation of the famous 1905 paper (A. Einstein, *Annalen der Physik*, 17, 891, 1905) in: H.A. Lorentz, A. Einstein, H. Minkowski and H. Weyl, *The Principle of Relativity - a Collection of Original Memoirs on the Special and General Theory of Relativity*, trans. W. Perrett and G.B. Jeffrey with Notes by A. Sommerfeld (London: Methuen, 1923), 35-65.

[20] Assuming, of course, the use of the same units of space and time.

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}} \text{ and } t' = \frac{t - \frac{v}{c^2}x}{\sqrt{1 - v^2/c^2}}$$

Here v is the constant velocity of frame F′ relative to frame F; and c is the measured speed of light. The reciprocal square root is frequently denoted by $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$.

IIB The Text

Figure 1[21] and paragraphs two and three of ‘Exploring the Twin Paradox’ (Tipler 1258-59).

The complete quotation is partitioned and indexed by T_1,T_2 T_3, etc.

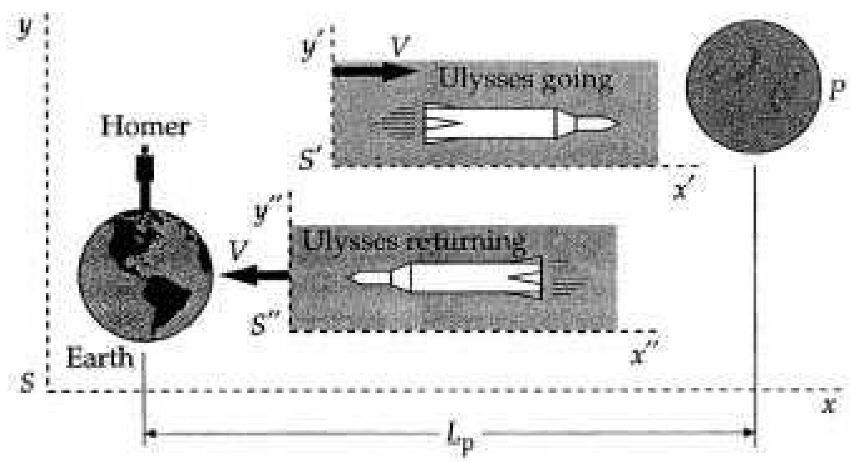

Figure 1 The twin paradox. The earth and a distant planet are fixed in frame S. Ulysses coasts in frame S’. His twin Homer stays on earth. When Ulysses returns, he is younger than his twin. The roles played by the twins are not symmetric. Homer remains in one inertial reference frame, but Ulysses must accelerate if he is to return home.

[21] The caption for Figure 1 also is from Tipler 1258.

Paragraph 2 of the text

T_1 Let the planet P and Homer on earth be at rest in reference frame S a distance L_P apart, as illustrated in Figure 1.

T_2 We neglect the motion of the earth.

T_3 Reference frames S′ and S′′ are moving with speed V toward and away from the planet, respectively.

T_4 Ulysses quickly accelerates to speed V, then coasts in S′ until he reaches the planet, where stops and is momentarily at rest in S. To return he quickly accelerates to speed V toward earth and then coasts in S′′, until he reaches earth, where he stops.

T_5 We can assume that the acceleration times are negligible compared with coasting times.

T_6 We use the following values for illustration: L_P = 8 light years and V = 0.8c. Then

$$\sqrt{1-V^2/c^2} = 3/5 \text{ and } \gamma = 5/3.$$

Paragraph 3 of the text

T_7 It is easy to analyze the problem from Homer's point of view on earth. According to Homer's clock, Ulysses coasts in S′ for a time $L_P/V = 10y$ and in S′′ for an equal time. Thus Homer is 20*y* older when Ulysses returns.

T_8 The time interval in S′ between Ulysses' leaving earth and his arriving at the planet is shorter because it is proper time. The time it takes to reach the planet by Ulysses' clock is $\Delta t' = \dfrac{\Delta t}{\gamma} = \dfrac{10y}{5/3} = 6y$. Since the same time is required for the return trip, Ulysses will have recorded 12*y* for the round trip and will be 8*y* younger than Homer upon his return.

IIIA Content (Hypothetical Expression)

Text used for each hypothetical expression is indicated in the parentheses.

A_1 (T_1, T_3, Figure 1) The reference frame is imagined to extend to the remote planet P. All three reference frames and their origins are imagined at once, and are represented in Figure 1.

A_2 (T_2, T_4, T_5, T_7) We consider the accelerations of the earth's rotation, the traveler's short launch and also the brief change of direction at the planet P to have relatively negligible effect on the calculations. For, in the main argument, the distance to the planet can be as large as we please. So whatever contributions to time and velocity might occur due to the short accelerations at the earth and at the planet P, they can be made relatively small compared to the long times and arbitrarily large distances of the journey at constant velocity V.

A_3 (T_6) From A_2, we neglect possible effects of accelerations at the beginning of the trip and at the planet P, and assume the constant velocity V relative to S is maintained at 0.8c. Then with L_P = 8 and using (Distance) = (Constant Velocity)(Time), we solve for the total time elapsed relative to S for the outward journey. This time is then found to be 10 y; and the same is obtained for the return journey. So the total time relative to S would be approximately 20*y*.

A_4 (T_3 , T_8) Let's assume that measurements made by Ulysses relative to suitable reference frames S′ and S′′ on the space ship give the same quantities as would be obtained by Homer on earth, if Homer were first to obtain measurements relative to frame S, and then use the transformation equations. Recall from the diagram that the primed and doubled primed coordinates refer to the outward and return journeys respectively. So, let's set up the notation for the calculation. For the measurements made by Homer (on earth and relative to frame S), we have $\Delta x_{outward}, \Delta t_{outward}$ and $\Delta x_{return}, \Delta t_{return}$. Let $\Delta x', \Delta t'$ and $\Delta x'', \Delta t''$ be the transformed quantities for the outward and return journey obtained by using the transformation equations on those S measurements respectively; let $\Delta\tilde{x}', \Delta\tilde{t}'$ and $\Delta\tilde{x}'', \Delta\tilde{t}''$ be the measurements obtained by the traveler Ulysses on the outward and return journeys, relative to S' and S'' respectively. Our assumption then can be written as follows: For the outward journey we would have $\Delta x' = \Delta\tilde{x}', \Delta t' = \Delta\tilde{t}'$; and likewise for the return journey we would have $\Delta x'' = \Delta\tilde{x}'', \Delta t' = \Delta\tilde{t}''$.

A_5 (T_8) Using the hypotheses of A_3 and A_4, we obtain $\gamma = \dfrac{1}{\sqrt{1 - V^2/c^2}} = 5/3$. The transformation equations then yield a total travel time for Ulysses to be the sum of

$\Delta t' = \dfrac{\Delta t_{outward}}{\gamma} = \dfrac{10}{5/3}$ and $\Delta t'' = \dfrac{\Delta t_{return}}{\gamma} = \dfrac{10}{5/3}$. For the two observers Homer and the traveler Ulysses, there would be, therefore, a difference in measured elapsed time. Specifically, the time measured by the traveler Ulysses (who is traveling at the large velocity of 0.8c) would be 8 years shorter than the measured time of the earth bound observer Homer.

IIIB Oversights

B_1 Regarding A_1: To begin, it is useful to recall that real reference frames do not extend as depicted in A_1. Indeed, even if instruments are attached to satellites, and even if there is a convenient way to imagine satellites as "out" in orbit, the actual data that finally enters into a real calculation would come from calibrated laboratory equipment. In any case, there are no visible axes extending outward into the solar system.

Certainly, within mathematics, and within the context of some metric geometry, one may define a "line" that "extends without bound". But even in mathematics, the basic datum for such conception consists of some imagined fragment of length. We return then to the fact that real reference frames are determined in laboratories, by finite data that are accessible to the laboratory scientist. In some cases, such data begin as actual laboratory lengths relative to some convenient ruler. More typically, however, even lengths are not measured in that direct way. For, as may be found in many undergraduate physics laboratories, there are networks of instrumentations (electronic, digital, etc) that provide plots (e.g. on screens) scaled relative to theoretically and experimentally justified interconnected sets of provisional standard units.

Note, finally, that where it is not possible to verify as imagined the prolongation "into space" of an imagined reference frame such as in A_1, even less so is it possible to verify as imagined three such imagined reference frames represented in Figure 1.

Evidently, however, diagrams and other images for reference frames can be eminently useful in both mathematics and physics. Further discussion of this issue will be left for Section IV.4 (below).

B_2 Regarding A_2 : The issue here is not that accelerations would have no effect on experimental results. The fundamental issue here concerns relative magnitude; and the meaning of the claim would seem to be compatible with the context. See, however, B_3.

B_3 Regarding A_3: The issue here is deceptively complex. It seems simple enough to hypothesize a constant velocity of V = 0.8c across a distance of 8 light years. But to what would this correspond in experiment? Any experiment will originate from some laboratory situation. A distance of 8 light years is not some imaginable distance as such. (See also B_1.) It can be defined to be the "distance" (what laboratory verified metric?) that light would "travel" (locally measurable radiation effects) in a locally measured time of 8 years. It is calculated therefore on the basis of verified local velocity experiments for light, hypotheses of special relativity (including that there is no measured distance without its measured time; and no measured time without its measured distance), and pertinent known results on standard units, etc. The speed of 0.8c is then defined in terms of c; the time of 10 years is hypothetical; and the distance of "10 light years" is tautological.

Is it possible for a spaceship or object to reach and sustain that velocity? Does the simple formula V = (Distance)/(Time) actually apply? If so, why the need for Einstein's results in the present context of special relativity, according to which any such calculation would require drawing in the hypotheses connected with synchronisation due to the fact that each location would have its own time. There are then questions and answers that would require experimental verification, and cannot be mathematically deduced as given in A_3 (T_6). Indeed, there are experiments that have been taken to provide evidence for time dilation, and so some further mention of experimental results will be made in Section IV.1.

With regard to Tipler's expression, it may be useful to observe that the calculations (e.g. from A_3 (T_6)) are of the old style, suggestive of an imaginable empty space, rather than a space-time continuum locally verified in concrete extensions and durations, as would be proper to the context of Special Relativity.

B_4 Regarding A_4: Measurements obtained by a traveler on the hypothesized space ship would be obtained using laboratory equipment on the ship. There are, therefore, two sets of measurements to consider:

(i) Lengths and times (x', t') relative to S′, as accessible to the experimenter on earth through the transformation equations applied to $(x_{outward}, t_{outward})$; and

(ii) Lengths and times $(\tilde{x}', \tilde{t}')$ relative to S′, as accessible to the experimenter on the space ship.

The derivation of the alleged time contraction uses the hypotheses of A_4, applied to measurements pertaining to both the outward and return parts of the space-ship journey. Completely similar remarks apply to both the outward and the return journey, so the present discussion only directly regards the outward journey. Now, for the hypotheses of A_4, one may ask on what grounds it may be assumed that transformed earth measurements be equal to the spaceship instrument measurements. The spaceship instruments, by hypothesis, are in their own frame, different from the earth frame. Can it simply be assumed that measurements from a remote moving laboratory location satisfy the claimed equality? Perhaps space-ship results can in some way be communicated to the earth experimenters? But, if any such communication occurs, it will necessarily make use of some further transmission data and transformation equations compatible with the hypotheses of relativity. For any such communication will be transmitted from what to the earth frame S is a moving apparatus, at some high velocity, at some remote location, at some remote time. To simply make the assumption that these further complications might not affect results is not only not consistent with experimental method, but breaks from the hypotheses of the context that is special relativity.

Even if some type of bi-data source were in some case obtained, above and beyond the usual laws of physics, the transformation problem would not be removed, but only be further complexified. For there would now be not one, but two sets of measurements (from two laboratory here-nows). Is one of these to take priority over the other? Or, by the principle of equivalence, are they to be considered equivalent, at least with regard to measurements? One could hardly revert to some non-verifiable notion of absolute space and time. So, there would still be a question of how one might correlate the measurements of one frame relative to the other. The present text, Tipler, however, is a physics text, and in physics we are not free to admit what is not empirically verifiable. Furthermore, from the results of special relativity, we cannot take space to be Euclidean and empty. Finally then, within the present context, there are no grounds for being able to identify the space-ship laboratory results with the transformed earth laboratory results.

Not all is lost. For from the earth laboratory, there might be the possibility of using some second set of measurements, calibrated in some way that would correspond with data originating from the moving spaceship, and measured perhaps relative to some differently calibrated set of measuring instruments S′ (units, etc.). In that case, there would be the possibility of comparing those laboratory measurements relative to S′ with the quantities obtained using the Lorentz transformation equations applied to the measurements (x, t) obtained relative to the first laboratory frame S. Note also that such use of the transformation equations would be consistent with Einstein's first postulate.

B_5 Regarding A_5: The calculation for A_5 makes use of A_3, A_4 and the transformation equations. The oversights of A_3, A_4 have already been considered. There is, though, a further fundamental insight that is in fact an incorrect use of the transformation equations.

As a preliminary to the mathematical details for discussing the use of the Lorentz equations, let's first consider an example that is more down to earth. Suppose then that one has two county maps, M and M′, and that (at least for a region surrounding a town), locations given by a pair (letter, number) from M are denoted by possibly different pairs (letter′, number′) from M′.(For example, M could be skew to M′; or M could be constructed using different units of land length; and so on.) In particular, suppose that the coordinates of the town using M are (E,4) say; and that the coordinates for the same town using M′ are (K′,10′).

Notice, that the correspondence between the two maps is a correspondence of pairs. Obviously, the fact that the one town is represented by two different pairs of coordinates does not imply that the coordinates individually agree. Indeed, in the present example, $\mathrm{E} \neq \mathrm{K}'$ and $4 \neq 10'$.

Now that other details have been attended to, let's look, at last, at Tipler's presentation of the traditional argument for time contraction. Suppose a displacement in distance and time where we take $x_1' = x_2''$. In Tipler this corresponds to the hypothesis that, relative to the reference frame on the space-ship, there is no change of location of the measuring instruments from one moment to the next. The transformation equations give that $\Delta t' = \dfrac{\Delta t}{\gamma}$. Hence, $\Delta t = \gamma \Delta t'$. Since $\gamma > 1$, it is said that the clocks in S′ run slower than the clocks in S.

When, however, $x_1' = x_2'$ and $t_2' - t_1' > 0$, the transformation equations correlate the length-time interval $\Delta s' = (\Delta x', \Delta t')$ with the length time interval (in the unprimed earth coordinates) $\Delta s = (\Delta x, \Delta t)$. For $\Delta x' = 0$, we then obtain that the length interval in S is given by $\Delta x = \Delta t' V \gamma$.

So, if (as in the derivation of the paradox) it is supposed that $x_1' = x_2'$ and $t_2' - t_1' > 0$, then relative to the earth frame there is also a corresponding change of location. In other words, the two times t_1 and t_2 that determine the time interval Δt relative to the earth frame S are two times at two different locations. Hence, by the fact that the transformation equations correlate pairs of coordinates and not single coordinates at a time, then just as with the illustration of the county maps, there is no basis in the transformation equations for identifying the time component intervals. Besides this mathematical error, we may also observe that by the original hypotheses of special relativity, times at different locations are simply not directly comparable.

IV Context of Concern in Physics

The sequence of topics in this section is not meant to be comprehensive. Instead, I briefly comment on what seem to me to be various key issues in physics that were implicit in my discussion of the Twin Paradox. Part of my background here includes efforts to read chapters I-V, VIII, XVI and XVII of *Insight*. There have been notable advances in physics in the last decades. So, where much of the explicit physics content of the present section is dated, in as much there is some validity to my comments, they would need to be brought into the modern context indicated by such books as O'Raifeartaigh,[22] Lawrie,[23] and Greiner.[24] There are leads on energy given in *Insight* that would need be followed up.[25] Moreover, all results would need to be taken up by the enriching controlling dynamic hodic process indicated by McShane's third word of metaphysics.

Note also that while my discussion focuses on physics pertaining to elementary things, eventually there will be the need for results that would regard the physics and geometries[26] of higher things. There are, then, the first and second words of metaphysics.[27] I think, however, that developments there probably will belong to future functional collaboration.

IV.1 Experiments on Time Dilation

In Section III, I referred to the fact that there have been various experiments that have been taken to provide evidence

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[22] Lochlainn O'Raifeartaigh, *The Dawning of Gauge Theory* (Princeton: Princeton UP, 1997).

[23] Ian D. Lawrie, *A Unified Grand Tour of Theoretical Physics* (Bristol and Philadelphia: Institute of Physics Publishing, 1990).

[24] Three (of many) books by Walter Greiner are: *Quantum Mechanics*, 2001; with J. Reinhardt, *Quantum Electrodynamics*, 2002; with S. Schramm and E. Stein, *Quantum Chromodynamics*, 2002 (all New York: Springer-Verlag). See also Dr. Greiner's Homepage: http://www.th.physik.uni-frankfurt.de/~greiner/.

[25] McShane enlarges the context in *Cantower* XXX

[26] See Sections IV.5 and IV.6 below.

[27] McShane, *Cantower* XXIV

for time dilation. So one might wonder about my conclusion that, within the context of Special Relativity, there are neither theoretical grounds nor experimental evidence for the so-called time dilation.

One of the early experiments that has been taken to give credence to time-dilation is discussed in detail in Swann.[28] My immediate purpose, however, is not to enter into all of the details of the experiment as such, but to point out where some of the conclusions represent what to my context does not seem consonant with a scientific point of view.

As I discussed already for the twins, to suppose what other experimenters "would be" measuring is neither grounded in scientific method nor in concord with the hypotheses of Special Relativity. Even "less verifiable" is the following: "…suppose that the lifetime of a mesotron (of velocity βc), as measured by one who accompanies it in its motion, is τ_0."[29] Even if it were possible to contradictorily both make use of and deny known laws of physics, and so accompany a mesotron (with a real massive laboratory frame and all of its apparatus and measuring devices, etc), the main objections that applied to Ulysses' space-ship would still apply. It is my view, therefore, that suppositions like these simply do not belong in the realm of natural science.

The cosmic ray experiments did reveal that "a mesotron with high energy has a lifetime which is greater (than a low energy mesotron) […] in the ratio" predicted using the scale factors $\sqrt{1-V^2/c^2}$ obtained from Einstein's equations.[30] The further inference, however, was that time for the high energy particles therefore slows down. But, the fact that relative to the calibrated instruments of the laboratory, an energized mesotron

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28 W.F.G. Swann, "Cosmic Rays," *Reports on Progress in Physics* (The Physical Society, London) Vol. X, 1944-45, 1-51. As mentioned by Lanczos, see especially p. 16. There are of course, more contemporary experimental results. For an extensive up-to-date bibliography, see Y. Z. Zhang, *Special Relativity and Its Experimental Foundations* (Singapore: World Scientific, 1997).

29 Swann 16, note †.

30 Swann 17.

tends to survive reactions in a cloud chamber measurably longer than a non-energized mesotron, suggests to me, not some non-verifiable time dilation (whose deduction in any case depends on flawed mathematics), but that the real differences in the energies of such particles correspond with measurable and statistically broader ranges of possible reactions. By analogy, a rapidly spinning top spins measurably longer before falling than a slower spinning top. But this says nothing about the rate of time relative to some imaginary and inaccessible observer traveling, as it were, on a top. Regarding the mesotrons, I suspect that what was partly revealed is the existence of verifiable connections between energy,[31] potency and the measurable life-times of such entities.

There remain the numerical results of the mesotron experiments, that is, that certain ratios obtained in experiment turned out to be approximately equal to what was calculated from Einstein's relativistic energy equations. These results (and more recent experiments referred to in Zhang's text) provide accumulating evidence that Einstein's postulates and theoretical developments were of reaching significance. As numerous alleged paradoxes show, however, understanding that significance is a further issue.

IV.2 The Complex Physics Context

Evidently, there are root problems influencing the proper development of physics. There is, for instance, a general acceptability of certain results whose theoretic conclusions are partly mixed with imaginative representation. But, when the merely imaginable is taken as scientific, the mesotron is then somehow "out" in "a space" that is empty. There is, it would seem, the necessary inconvenience of having to deal with issues of synchronization (and in General Relativity, the bending of light rays around massive objects). Besides such oddities, however, if only we had better instruments, we would be able to see electrons, mesotrons, quarks, and the like, to be the little grains or imaginable wavicles, wavelets, strings, knots or surfaces that they really are. Note, however, that even when description is empirical (as when for example, an investigator

[31] McShane, *Cantower* XXX.

examines tracks on a laboratory photo-plate, or a person inspects images of the night sky), these images as images are not the explanatory correlations that the scientific investigator normally works so hard to verify.

This problem of description versus scientific understanding does not point to some merely-philosophic issue extraneous to progress in physics. For, as is now historically evident, lacking control of meaning can both over-turn and suppress postulates, admit the non-verifiable and the contradictory, and even allow for basic mathematical errors to be consistently ignored.[32] Other examples can be easily found. For instance, it is still taught in graduate schools that, besides measurement and statistical difficulties usually associated with the Heisenberg Uncertainty Principle, electrons and other subatomic particles orbit central cores or "nuclei". Known laws have prohibited that as a real possibility for more than 100 years.

I certainly don't mean to suggest that these problems are simple or are to be easily resolved. It seems to me, rather, that part of the difficulty is that the context of theoretical physics is so complex. Whatever one's allegiance or philosophy of science, it cannot be denied that work in physics involves imagination and description; measurement; definitions and equations; frequencies, abstract postulates, identifications, and more. So, to not be engaged in an increasingly precise control of these operations is bound to lead to endless confusion and alleged paradox. What physics needs, then, is a methodical division of labour, as discovered by Lonergan, and as sketched out for mathematics.[33] This would gradually reveal critical flaws and move the group to a fuller control of meaning.

IV.3 Invariance

The question of invariance does not normally arise in the higher (and more difficult) sciences. In physics, however, we use measuring techniques based on the best available standard

[32] With regard to the Twin Paradox, the mistake in the use of the transformation formulas has remained in acceptance since at least 1911. Ref. Einstein's paper: see note 2.

[33] See note 8, above.

units of the day, corresponding laboratory reference frames and coordinate systems. So, in physics, defining correlations would necessarily be invariant under actually possible changes of reference frame that, according to the best available theoretical understanding also are likewise mathematically permissible. In the Theory of Special Relativity, permissible changes of reference frame are determined by the Lorentz group. This provides a mathematical range of possibilities. But which of these changes of frame are actually possible is not a question for mere mathematical deduction, but calls for experimental verification in actual electromagnetic phenomena.

In the metaphysics of physics, a term-defining correlation that is suitably invariant would be called a primary relation. It would be the (explanatory) relation that would be generally verifiable. But in physics, seeking primary relations reveals other features of the process. For, where convenient scales might be used to approximately verify a specific combination of measured ratios, an investigator need not point out that the particular ratios in one case are different from those in another. The focus, rather, can be on the combination of ratios, not the particular ratios as such. For a primary relation in physics then, what the particular quantities are at a given time and at a given location would be secondary.[34]

It is interesting to advert to these distinctions in connection with Einstein's two postulates of Special Relativity. Restricted invariance was already a topic in Newtonian physics; and the constant speed of light played a role in efforts to understand Maxwell's equations. But, to raise invariance to the status of a postulate was an enormous move forward for physics. In Special Relativity, that postulate evidently expresses Einstein's implicit grasp and (special) breakthrough that physics seeks (suitably invariant) relations. The second postulate, while grounded in experimental results, is quite different in significance. For, in as much as terms and units survive in an explanatory context, a measured speed is some kind of approximation toward secondary determination. So, while we may find the postulate on the speed of light to be generally

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[34] *CWL 3*, Ch. 16.

verified, there would seem to be no principle of empirical science that could rule out the possibility that the measured speed of light relative to inertial frames might not vary; or perhaps change with the age and state of the universe.

IV.4 Minkowski Space, World Lines and Logic

Frequently, texts on Special Relativity include some mention of "world lines". Space-time axes are drawn, the coordinates of a trajectory are combined, graphs are formed relative to these axes. Notice, however, that this is mathematical construction. The "world-lines" are imagined trajectories relative to imagined scaled axes that are imagined to be "perpendicular". Certainly, images play a role in physics, as does mathematical creativity. The problem that I look to now is the possible physical status of what for imagined world lines often is taken to be the ambient "Minkowski space".

The set-up called "Minkowski space" is obtained as follows: Following Einstein's idea, each measured location has its own measured time. As is customary, suppose a ruler of standard material is scaled with some standard length, and that standard clocks are located at each standard unit distance along the ruler. Altogether, the scaling and the clocks determine a coordinate frame. One may then use the Minkowski distance formula $\Delta s^2 = \Delta x^2 - \Delta t^2$ to define "distance" between location-times of this frame.

On the mathematical side, one may define distance in this way, between points defined to be elements of a coordinate space. In as much as the images for this include imagined extensions and durations, one may even call this a "geometry". But that is mathematics. For physics, there is the hypothesis that relative to a stationary origin, relatively stationary axes of a coordinate frame can be used to unambiguously give locations and instants. There also is the key hypothesis that the frame itself does not significantly add to the physics of the situation. Finally, the Lorentz coordinates, by definition, refer to no empirical extension or duration of any physical process (other than the constructed frame itself). In other words, "Minkowski space" refers to a situation where, by definition, nothing is going on. And since this does not regard trajectories of physical processes as such, the experimental fact is that there is no evidence for a real "Minkowski space".

I mean that last paragraph as a help, by way of exclusion, toward determining a possible physical significance of the Minkowski "metric". For, going on the progress of physics over the last century, there is undoubtedly significance to Minkowki's approach. The question then is, if the construction does not yield a geometry, what does it yield? For this, I look to how the construction is used in practice. But there, the invariance of the metric is used as the criterion for permissible changes of coordinate frames. In other words, the invariance of the metric determines ranges of possible universes of discourse. It seems to me therefore, that where Minkowski's approach does not yield a geometry as such, it does pertain to the Logical Note of Section V.2.6 of *Insight*.

IV.5 Space, Time, Real Geometries and the Dynamical Universe

The extensions and durations investigated in physics are concrete. So, recalling that the word "geometry" comes from the Greek words for "earth" and "measure", perhaps it is not unreasonable to call the objective of physics "real geometry". Moreover, doing so actually helps point to a further and central component to the physics project, one on which I have not yet commented. For lengths and times are not "things" but are "of things". So, in physics, we also seek the identities of things that ground and unify probably verified conjugate forms. We approach then a scientific notion of "space-time-as-explained" that leaves no room for the imagined empty space and general time usually associated with the work of Newton (but that also implicitly continues to intrude on contemporary work).

Frequently, experiments take place under rather exceptional circumstances - of say a laboratory a mile or more underground. Specialized experiments may help investigators discover structures and deduce possible schemes. But the very fact that investigators need to go to such lengths to isolate their experiments implicitly acknowledges a dynamic propensity in things and concretely provides on-going evidence of real randomness.[35] Moreover, whatever the ultimate account, the

[35] See *CWL 3*, II.4; and McShane, *Randomness, Statistics and*

success of the multi-Lagrangian Standard Model suggests that there are distinguishable networks (or perhaps “groupings”) of species of elementary things, that taken together can re-act, “form integral compounds,”[36] partially “dis-integrate”, be created and annihilated, in dynamic patterns revealing of and constitutive of something perhaps akin to a periodic table. So, even when a full account might be reached of all possible types of elementary geometric-physical entity, there will remain the non-systematically occurring particle sequences revealing controlling geometric forms and dynamic propensities of the non-static universe.

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Emergence (Dublin: Cahill, 1970).

[36] Regarding the meanings of “formed” and “aggreformic” see *CWL 3,* VIII.6; McShane, *Randomness, Statistics and Emergence;* and McShane, *Cantower* XXIX.