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Review of Noah Horwitz, *Reality in the Name of God* (Createspace, 2012). 358 pages.

Before anything else, I must state that this is a fascinating book, and I highly recommend it to the reader who is interested in the thesis that nature is in some important sense an information processor, a thesis which I heartily endorse. The book is indeed thought provoking and the author consistently displays courageous ambition, both with respect to the scope of his topic, and in his deep engagement with the loftiest of ideas.

The book consists of four chapters bracketed by an introduction and epilogue. The first chapter is a reasonably thorough survey of 20th century theology. The second chapter, which will be the focus of the critical aspects of this review, deals primarily with the relationship between set theory and ontology, with a focus on a dialogue with the atheistic and set-theoretically laden philosophy of Alain Badiou. The third chapter deals with what can only be called the Pythagorean understanding of creation (i.e., that all is number) and the modern concept of information as it applies to ontology. The fourth chapter is a somewhat aggregative, yet highly stimulating discourse on topics varying from the freedom of the will to the resurrection of the dead. At the end of the day, I have no choice but to admit that Horwitz is a heavy hitter of the first order. All criticisms aside, I admire his brave and sincere engagement with the deepest of all topics: theology, ontology and mathematics.

There is enough material in Horwitz's book that a review of it could itself form the content of an entire book. However, book reviews must be limited in length, so I must cut my remarks down to an appropriate size, and address what I take to be the key mistake in Horwitz's discussion: namely that the Kabbala has anything to do with Cantor's understanding of the relation between God and creation as mediated by mathematics.

As was the case with Ludwig Wittgenstein and Cantor's arch-nemesis Leopold Kronecker, the deep racism of 19th and 20th century European culture has led far too many to misinterpret Cantor's intentions in his use of the Hebrew alphabet in the notation of the transfinite arithmetic. Like Wittgenstein, Cantor was ethnically descended from Jews; however, his family had converted to Catholicism long before he was born. Cantor's Catholic faith is best demonstrated in his letters to the Pope of his day.

In making this rather controversial severance of Cantor from the Kabbalist tradition, surely the first question the reader will ask is: why did the Hebrew alphabet figure so prominently in Cantor's notation? To answer this, we must roll up our sleeves and engage with the nitty-gritty of Cantor's transfinite arithmetic. According to Kit Fine,[1] Cantor understood that every set is given to the mind with a particular order. A set is said to be well-ordered when each of its non-empty subsets has a first member. An ordinal number is the logical abstraction of the form of a well-ordered set. A cardinal number is the logical abstraction of an ordinal number, and hence is the result of a double act of abstraction, insofar as it is presented to the human mind. Two sets have the same ordinal number when they can be put into one-to-one correspondence in such a way that the first member of each subset of the domain is mapped to the first member of each subset of the image. Two sets have the same cardinal number simply when they can be put into one-to-one correspondence.

Now the distinction between ordinal and cardinal numbers can only be properly understood in the context of infinite sets. Given a finite set, there is exactly one ordinal number which is possible for this set to manifest. For example,

1, 2, 3

has the same order-form as

3, 1, 2

The same cannot be said, however, for infinite sets, as the following will illustrate:

0, 1, 2, 3, . . . ($m \prec n$ if and only if $m < n$)

0, 2, 4, . . . , 1, 3, 5. . . ($m \prec n$ if and only if m is even and n is odd or $m < n$)

Note that the two examples above involve distinct orderings of one and the same set, and consequently can be put into one-to-one correspondence (namely, identity); however, no such correspondence will preserve the order. So an infinite set has a unique cardinal number, but can have many ordinal numbers.

In this respect, the cardinal number is to the ordinal number what the clay is to the statue. The cardinal number forms the logical raw material which can be put into a variety of distinct orders. The cardinal numbers are, therefore, the foundation on which the ordinal numbers are to be built.

This is the reason why Cantor chose to denote the infinite cardinals with letters of the Hebrew alphabet, and the ordinal numbers with the letters of the Greek alphabet: because the Old Testament (i.e., the *foundation*) is written in the Hebrew alphabet, whereas the New Testament is written in Greek. Horwitz, along with far too many others, emphasizes the cardinal numbers at the expense of failing to mention the ordinal numbers, which were far more central to Cantor's theory. In

[1] "Cantorian Abstraction: A Defense and Reconstruction," *Journal of Philosophy* XCV 12, 599–634 (1998).

fact, most set-theorists today regard the cardinal numbers as dispensable, since the axiom of choice guarantees that each cardinal has a first, or least ordinal, which can stand proxy for the cardinal itself.

Horwitz's engagement with the primacy of the empty set, the set with no members, is both fascinating and commendable. However, he neglects to diligently respect a fundamental division which is perhaps the driving force of western thought since Parmenides up to the current day, which we illustrate with a table:

Pythagoras	intellect	sensation
Plato	form	particular
St. Paul	spirit	flesh
Descartes	mind	body
Cantor	set	member

The empty set, the one that is, must be distinguished from its extension, which is not. No set can be its own member: this is the lesson of Russell's famous paradox, which is simply a watering down of a paradox well-known to Cantor for digestion by the masses, namely, the paradox of the greatest ordinal. Any set of ordinal numbers is itself well-ordered, and hence has an ordinal number. So the totality of ordinal numbers, which Cantor denoted by Ω, cannot be a set, since if it were, it would have an ordinal number, which can be exceeded by adding one, which thereby results in a contradiction, since the order-type of the totality of ordinals must be greater than every particular ordinal.

This is the key to understanding Cantor's view on the relation of transfinite arithmetic to God as the absolute infinite which cannot be exceeded (see Ecclesiastes 7:14). The totality of ordinals is what Cantor called an inconsistent multiplicity: that is, the supreme infinitude of God cannot be captured by the limited logic of man, since to conceive of the totality of ordinals as a set results in a contradiction (as is the case with the totality of sets in the language of Russell's paradox.) Yet, the human mind knows an ordinal number, or for that matter a set, when it sees one. So man is thereby humbled before the supreme inexhaustibility of the absolute infinite of theology, where God acts as the limit (*peras*) of the transfinite cardinals and ordinals.

In his defense, Horwitz comes very close indeed to capturing this idea by footnoting Isaiah 55: 8: "My thoughts are not your thoughts." In conclusion, I do not condemn Horwitz's book in any way, as we are all limited creatures who can only strive for, but never reach, perfection. In this regard, I applaud Horwitz's efforts to strive in this manner, and I highly recommend his book. But I completely denounce any attempt to fuse Cantor's thought with Kabbala.

Stephen Gardner